Microwave Circuits and Antenna Design

Transmission Lines
&
Smith Chart

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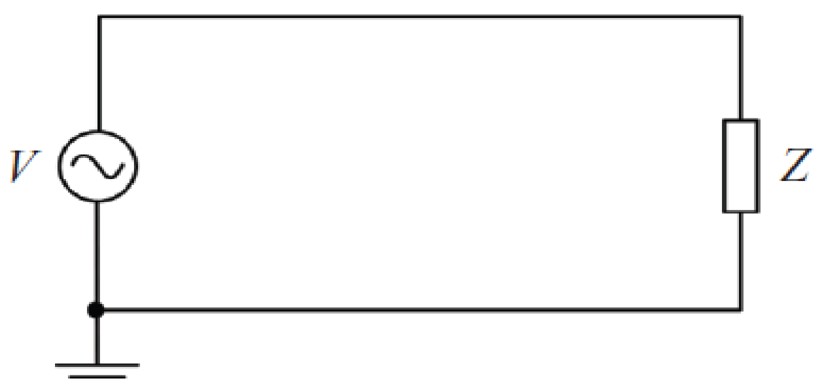
2.1 Circuit Concepts

- ***Electric current*** *I* is a measure of the charge flow/ movement.
- ***Voltage*** *V* is the difference of electrical potential between two points of an electrical or electronic circuit.
- ***Impedance*** *Z = R + jX* is a measure of opposition to an electric current.



$$I = \frac{V}{Z}$$

$$P = IV = V^2/R = RI^2 \text{ for DC}$$

$$P_{av} = \frac{1}{2} I_0 V_0 = \frac{V_0^2}{2R} = \frac{1}{2} R I_0^2 \quad \text{for AC}$$

Lumped and Distributed Element Systems

- The current and voltage along a transmission line may be considered unchanged (which normally means the frequency is very low). The system is called a ***lumped element system***.

- The current and voltage along a transmission line are functions of the distance from the source (which normally means the frequency is high), thus the system is called a ***distributed element system***.

2.2 Transmission Line Theory

- A ***transmission line*** is the structure that forms all or part of a path from one place to another for directing the transmission of energy, such as electrical power transmission and microwaves.

- We are only interested in the transmission lines for RF engineering and antenna applications. Thus dielectric transmission lines such as optical fibres are not considered.

Quality Factor and Bandwidth

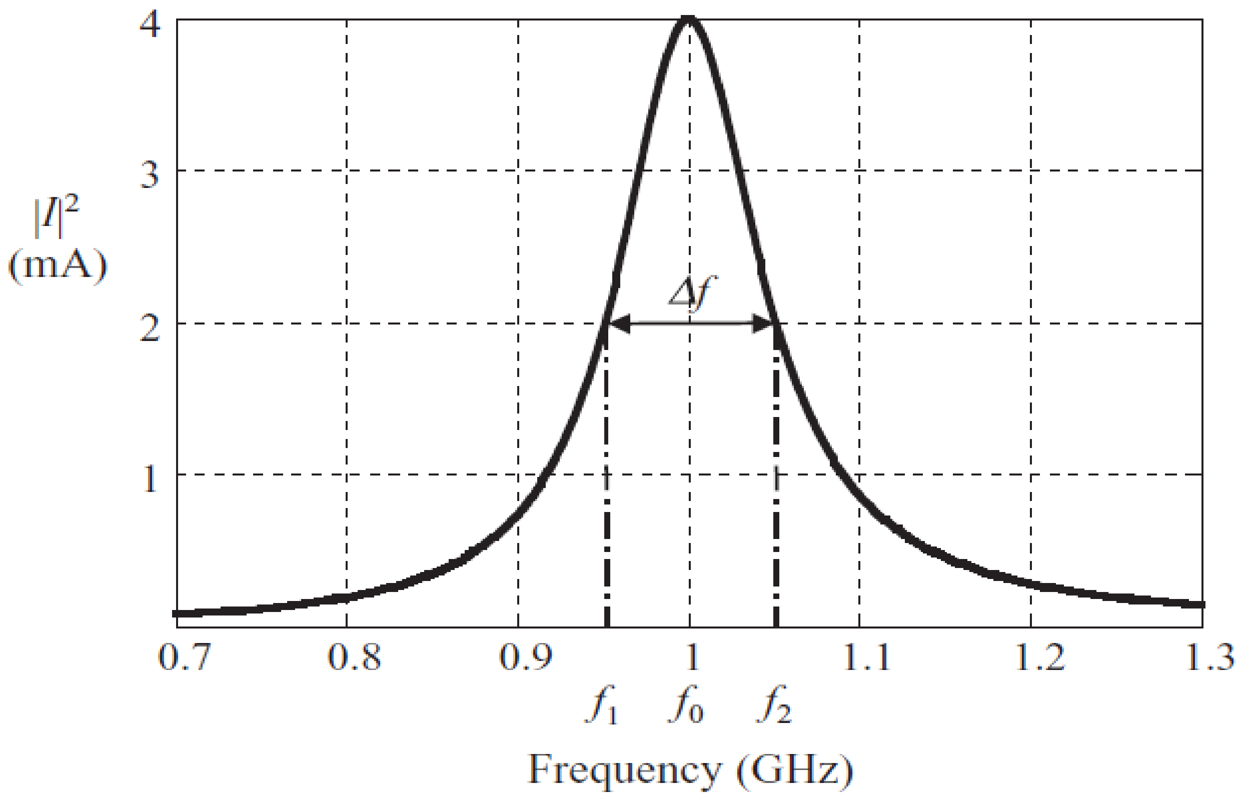
- ***Quality factor, Q***, which is a measure of how much lossless reactive energy is stored in a circuit compared to the average power dissipated.

$$Q \equiv \omega \frac{\text{(total energy stored)}}{\text{(average power loss in the load)}} = \omega \frac{W_E + W_M}{P_L}$$

where W_E is the energy stored in the electric field, W_M is the energy stored in the magnetic field and *PL* is the average power delivered to the load.

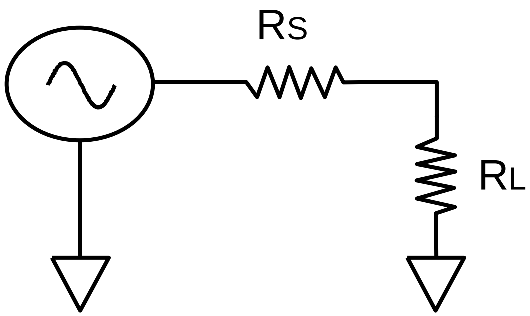
- *Antennas are designed to have a low Q, whereas circuit components are designed for a high Q.*

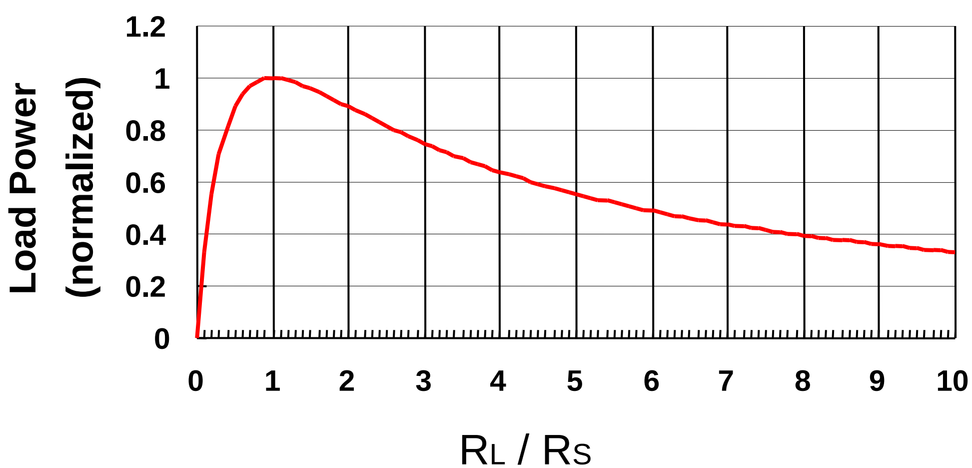
$$Q_0 = \frac{f_0}{f_2 - f_1} = \frac{1}{B_F}$$



where f_1 and f_2 are the frequencies at which the power reduces to half of its maximum value at the resonant frequency, f_0 and where B_F is the ***fractional bandwidth***. This relation only truly applies to simple (unloaded single resonant) circuits.

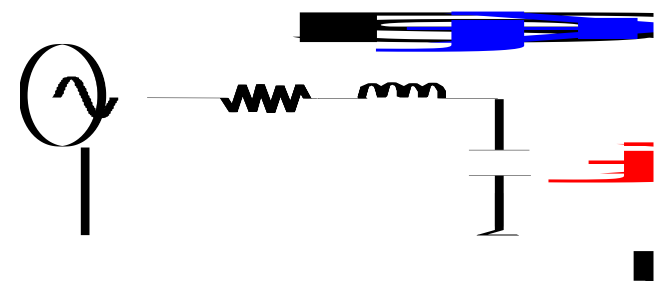
Power Transfer Efficiency





Maximum power is transferred when $R_L = R_S$

For complex impedances, maximum power transfer occurs when $Z_L = Z_S^*$ (conjugate match)



Transmission Line

Transmission Lines

- Since no lumped elements like resistors, capacitors are used at microwave frequencies, only transmission lines are used. Hence they are called distributed parameter network.

- Characteristic impedance is defined as that impedance of a line which is a constant when measured at any point on the line, impedance which is a constant at any point on the transmission line.

- Propagation constant γ is given by: $\alpha+j\beta$. Propagation constant is a complex sum of α and β, α being the real value and β being the complex part.

Transmission lines

- Transmission line types
  - All depend on electromagnetic phenomena
    Electric fields, magnetic fields, currents
  - EM analysis tells us about
    α attenuation vs. frequency
    β propagation velocity vs. frequency
    Z_o characteristic impedance
    relative dimensions
  - Parameters contributing to these characteristics
    σ conductivity of metals
    ε_r' real part of relative permittivity
    ε_r'' imaginary part of relative permittivity
    μ_r relative permaebility (usually ~ 1)

    structure dimensions
    h height
    W width
    L length
    t thickness

Transmission lines

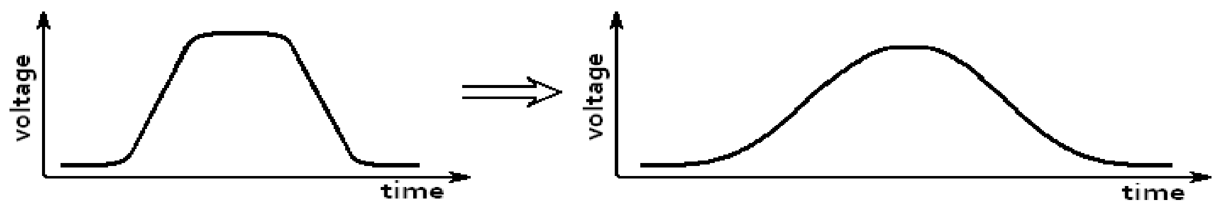
–Effects of transmission line parameters on system performance

–Attenuation vs. frequency

Attenuation – reduces signal amplitude, reduces noise margin
Freq-dependent attenuation – reduces higher frequency components, increasing T_r

–Propagation velocity vs. frequency

velocity – determines propagation delay between components
reduces max operating frequency
Freq-dependent velocity – distorts signal shape (dispersion)
may broaden the pulse duration

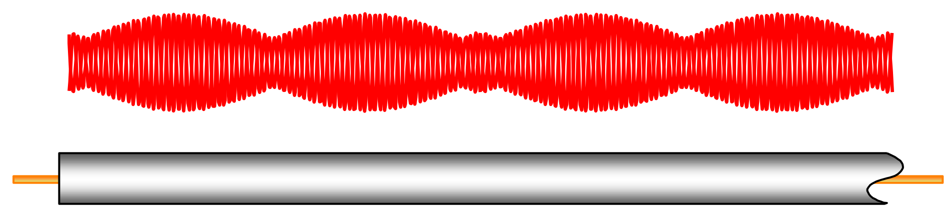


Characteristic impedance – ratio of V/I or E/H
determines drive requirements
relates to electromagnetic interference (EMI)
mismatches lead to signal reflections

Transmission Line Basics

Low frequencies

- wavelengths >> wire length
- current (I) travels down wires easily for efficient power transmission
- measured voltage and current not dependent on position along wire

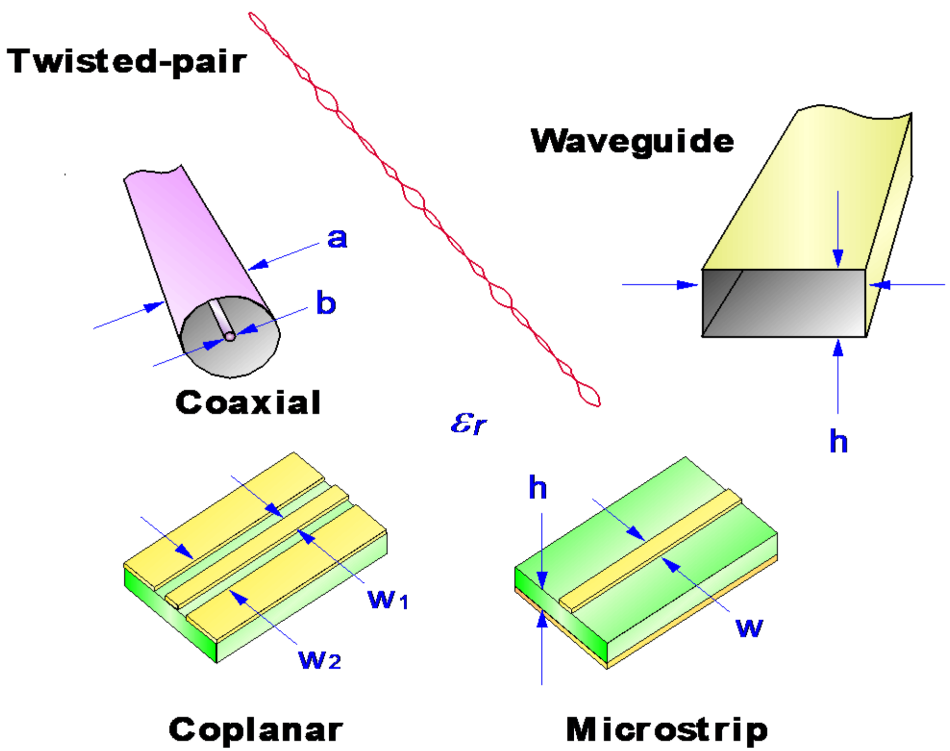


High frequencies

- wavelength ≈ or << length of transmission medium
- need transmission lines for efficient power transmission
- matching to characteristic impedance (Z_o) is very important for low reflection and maximum power transfer
- measured envelope voltage dependent on position along line

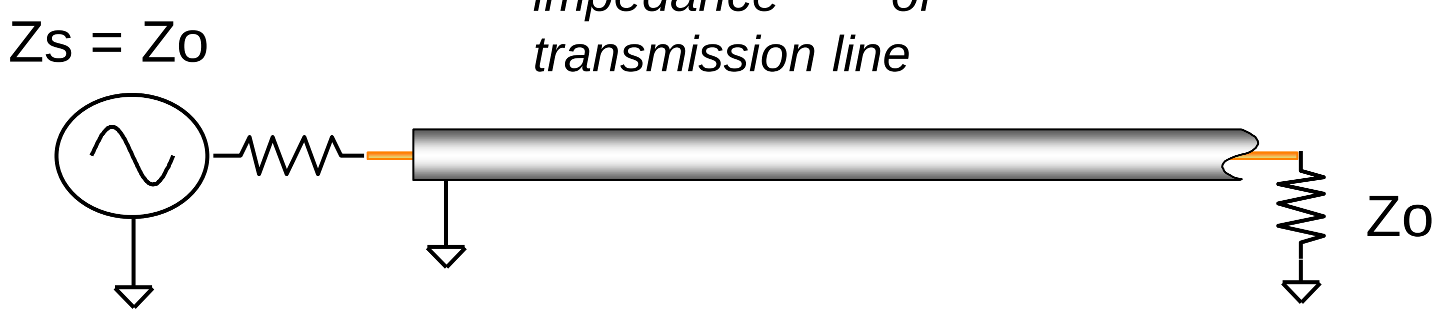
Transmission line Zo

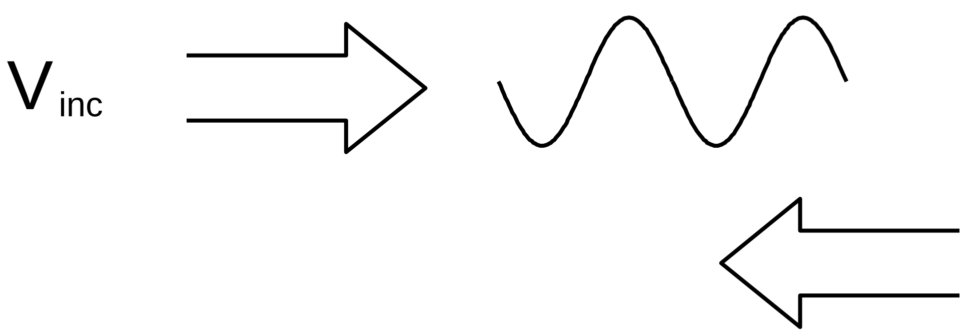
- Zo determines relationship between voltage and current waves
- Zo is a function of physical dimensions and ε_r
- Zo is usually a real impedance (e.g. 50 or 75 ohms)



Transmission Line Terminated with Zo

Zo = characteristic impedance of transmission line

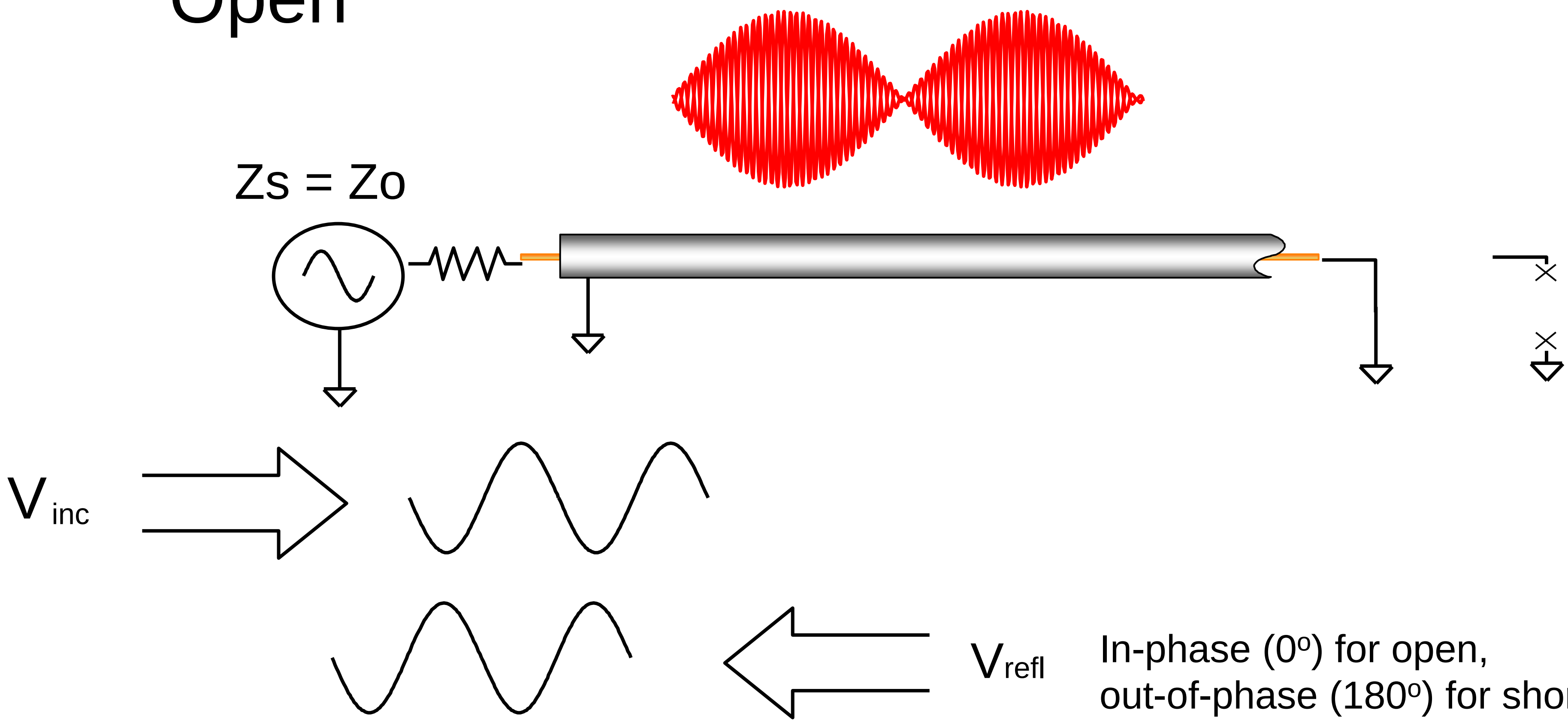




V refl = 0! *(all the incident power is absorbed in the load)*

For reflection, a transmission line terminated in Zo behaves like an infinitely long transmission line

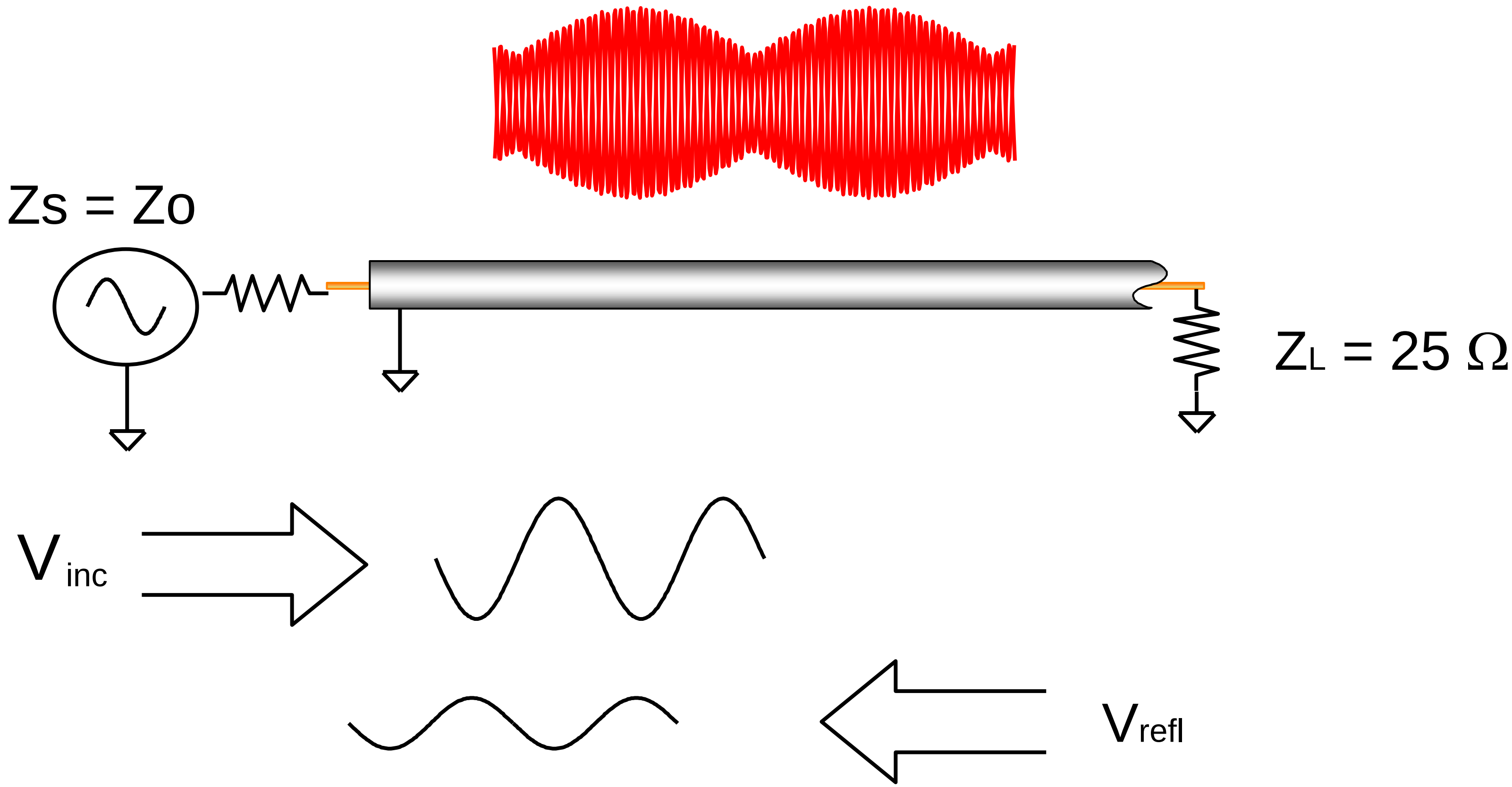
Transmission Line Terminated with Short, Open



In-phase (0^o) for open, out-of-phase (180^o) for short

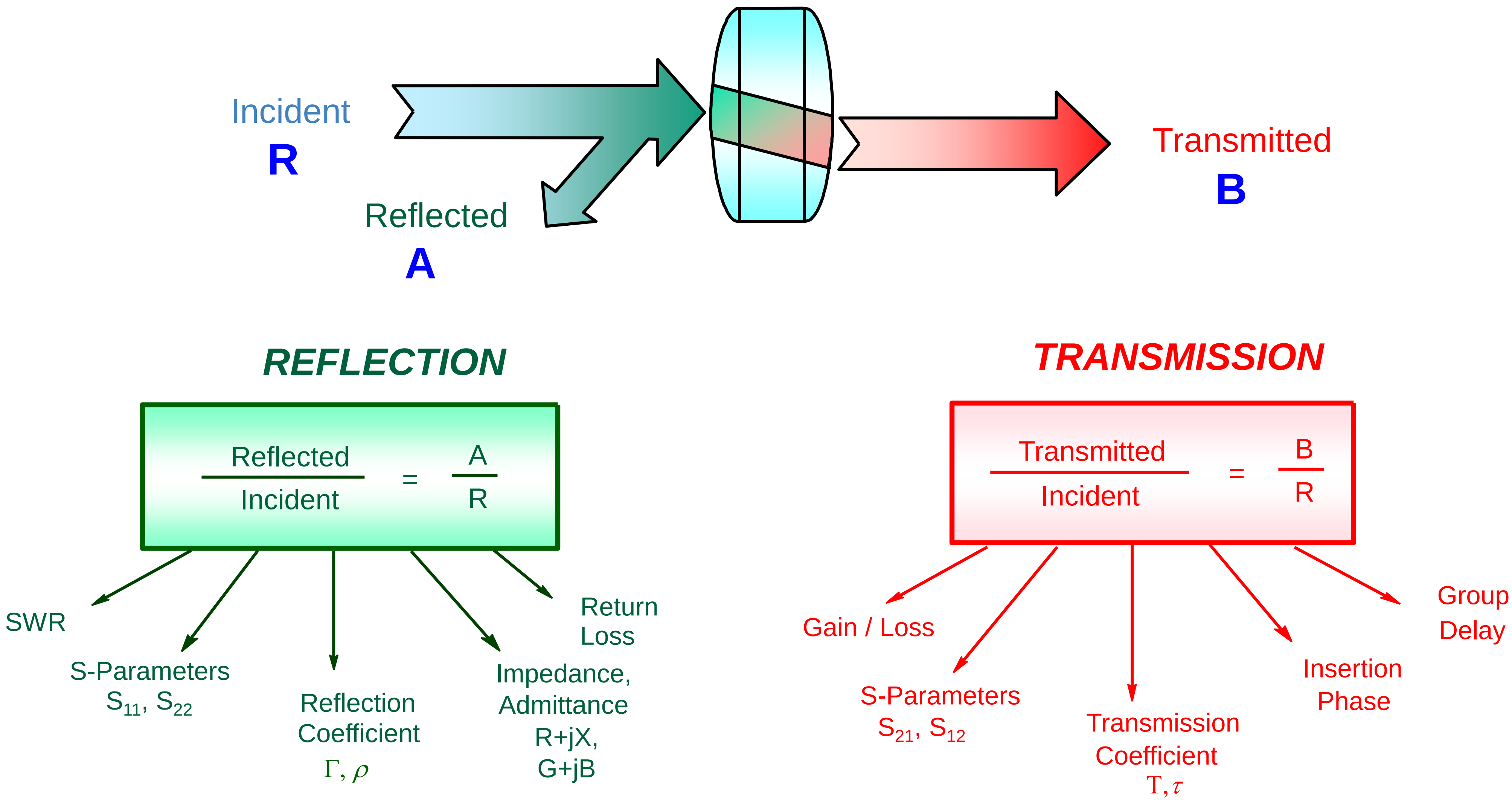
For reflection, a transmission line terminated in a short or open reflects all power back to source

Transmission Line Terminated with 25 Ω



Standing wave pattern does not go to zero as with short or open

High-Frequency Device Characterization

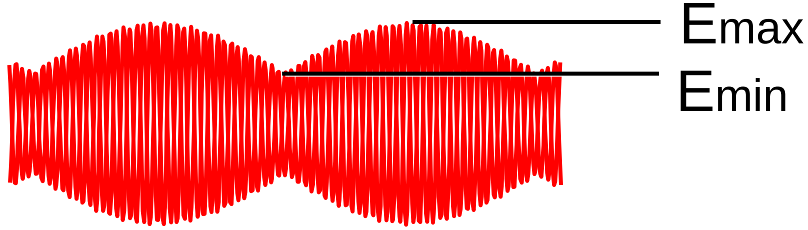


Reflection Parameters

Reflection Coefficient

$$\Gamma = \frac{V_{reflected}}{V_{incident}} = \rho \angle \Phi = \frac{Z_L - Z_O}{Z_L + Z_O}$$

Return loss = -20 log(ρ), $\rho = |\Gamma|$



Voltage Standing Wave Ratio

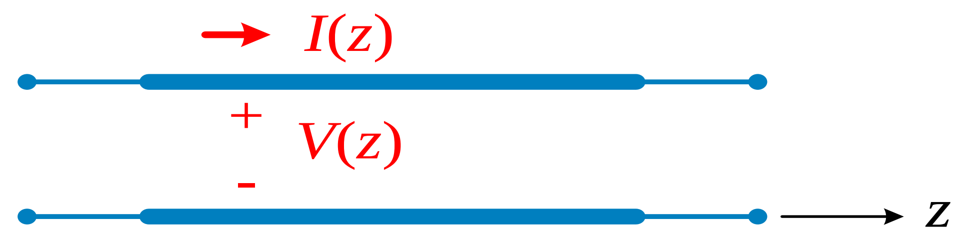
$$\textbf{VSWR} = \frac{E_{max}}{E_{min}} = \frac{1+\rho}{1-\rho}$$

No reflection
(Z_L = Zo)

Full reflection
(Z_L = open, short)

0	ρ	1
∞ dB	**RL**	0 dB
1	**VSWR**	∞

Basic TL formulas



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

$$\gamma = \alpha + j\beta = \left[(R + j\omega L)(G + j\omega C) \right]^{1/2}$$

$$Z_0 = \left(\frac{R + j\omega L}{G + j\omega C} \right)^{1/2}$$

guided wavelength $\equiv \lambda_g$

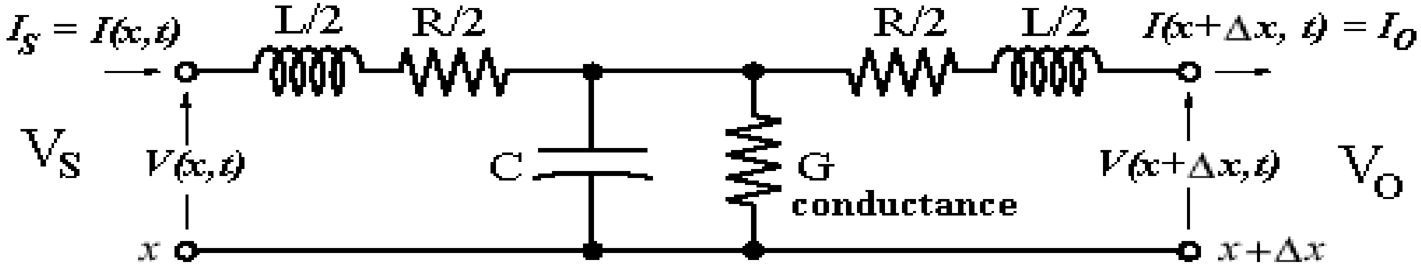
$$\lambda_g = \frac{2\pi}{\beta} \, [\mathrm{m}]$$

phase velocity $\equiv v_p$

$$v_p = \frac{\omega}{\beta} \, [\mathrm{m/s}]$$

Transmission line modeling

- Circuit model for incremental length of transmission line



–It can be shown that for a sinusoidal signal, $\omega = 2\pi f$, $V_s = A\, e^{j(\omega t+\phi)}$

$$V_O / V_S = e^{-\gamma z} \quad \text{propagation along z axis}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

–is analogous to $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$ in plane – wave propagation

–where γ is complex, $\gamma = \alpha + j\beta$ is the *propagation constant*

–the real part, α is the *attenuation constant* [Np / m : Np = Nepers]

–the imaginary part, β, is the *phase constant* [rad / m : rad = radians]

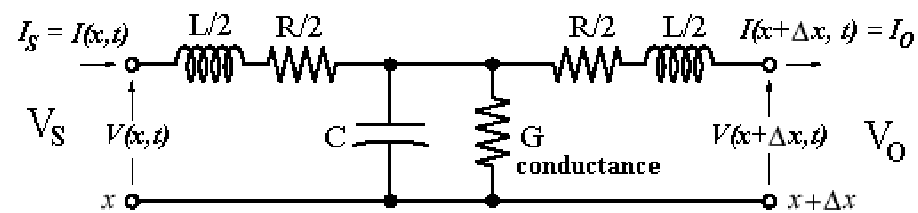
$$V_O / V_S = e^{-\alpha z}\ e^{-j\beta z}$$

amplitude change

phase change

Transmission line modeling

- Characteristic impedance, Z_o
  - ratio of voltage to current, $V/I = Z$



  - From transmission line model

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

  - typically G (conductance) is very small ≈ 0, so

$$Z_o = \sqrt{\frac{R + j\omega L}{j\omega C}}$$

  - At low frequencies, $R >> \omega L$ when $f << R/(2\pi L)$ the transmission line behaves as an R-C circuit

Z_o is complex
Z_o is frequency dependent

$$Z_o \cong \sqrt{\frac{R}{j\omega C}}$$

  - At high frequencies, $\omega L >> R$ when $f >> R/(2\pi L)$

Z_o is real
Z_o is frequency independent

$$Z_o \cong \sqrt{\frac{L}{C}}$$

Lossless Case

$$R = 0, \;\; G = 0$$

$$\gamma = \alpha + j\beta = \left[(\cancel{R} + j\omega L)(\cancel{G} + j\omega C) \right]^{1/2}$$
$$= j\omega\sqrt{LC}$$

so

$$\alpha = 0$$
$$\beta = \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{1}{\sqrt{LC}}$$

(indep. of freq.)

$$Z_0 = \left(\frac{\cancel{R} + j\omega L}{\cancel{G} + j\omega C} \right)^{1/2} \Rightarrow Z_0 = \sqrt{\frac{L}{C}}$$

(real and indep. of freq.)

Lossless Case (cont.)

$$v_p = \frac{1}{\sqrt{LC}}$$

In the medium between the two conductors is homogeneous (uniform) and is characterized by (ε, μ), then we have that

$$LC = \mu\varepsilon$$

The speed of light in a dielectric medium is

$$c_d = \frac{1}{\sqrt{\mu\varepsilon}}$$

Hence, we have that

$$\boxed{v_p = c_d}$$

The phase velocity does not depend on the frequency, and it is always the speed of light (in the material).

Propagation Constant

1) The manipulation of complex numbers has been discussed. Derive the formula for attenuation, α, as shown below. Given $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$ and $\gamma = \alpha + j\beta$; use $\operatorname{Re}\gamma^2 = \alpha^2 - \beta^2$ and $|\gamma^2| = \alpha^2 + \beta^2$ to generate the following formula for attenuation:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

Use MatLAB to generate a log-log plot of this attenuation for "dry" earth for frequencies from 100 Hz to 100 GHz where earth has a conductivity of 10^{-3} S/m and ε_r of 8.

Lossy media $\Rightarrow$ $(\sigma>0,\ \mu=\mu_r\mu_o,\ \epsilon=\epsilon_o\epsilon_r)$

$$c_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\cdot 10^8 \text{ [m/s]} \quad \text{(free space)}$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta \quad \text{(complex)}$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_r\varepsilon_0}} = \frac{1}{\sqrt{\mu_0\varepsilon}} \quad \text{(dielectric)}$$

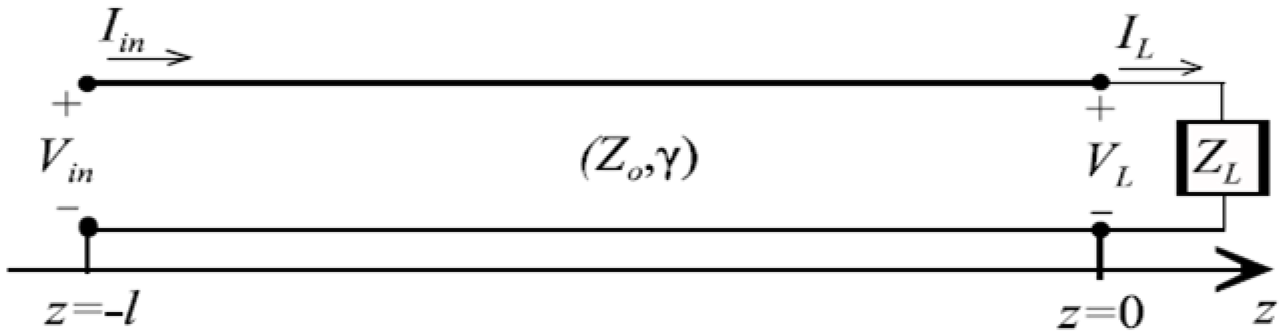
$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]} \qquad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

SPECIAL CASE OF LOSSLESS TRANSMISSION LINES

TABLE 2.2 Formulas for Transmission Lines

Quantity	General Line	Lossless Line
Propagation constant, $\gamma = \alpha + j\beta$	$\sqrt{(R + j\omega L)(G + j\omega C)}$	$j\omega\sqrt{LC}$
Phase constant, β	$\mathrm{Im}(\gamma)$	$\omega\sqrt{LC} = \frac{w}{v} = \frac{2\pi}{\lambda}$
Attenuation constant, α	$\mathrm{Re}(\gamma)$	0
Characteristic impedance, Z_0	$\sqrt{\frac{R + j\omega L}{G + j\omega C}}$	$\sqrt{\frac{L}{C}}$
Input impedance, Z_{in}	$Z_0 \frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l}$	$Z_0 \frac{Z_L \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jZ_L \sin \beta l}$
Impedance of shorted line	$Z_0 \tanh \gamma l$	$jZ_0 \tan \beta l$
Impedance of open line	$Z_0 \coth \gamma l$	$-jZ_0 \cot \beta l$
Impedance of quarter-wave line	$Z_0 \frac{Z_L \sinh \alpha l + Z_0 \cosh \alpha l}{Z_0 \sinh \alpha l + Z_L \cosh \alpha l}$	$\frac{Z_0^2}{Z_L}$
Impedance of half-wave line	$Z_0 \frac{Z_L \cosh \alpha l + Z_0 \sinh \alpha l}{Z_0 \cosh \alpha l + Z_L \sinh \alpha l}$	Z_L
Reflection coefficient, Γ_L	$\frac{Z_L - Z_0}{Z_L + Z_0}$	$\frac{Z_L - Z_0}{Z_L + Z_0}$
Voltage standing-wave ratio (VSWR)	$\frac{1 + \lvert\Gamma_L\rvert}{1 - \lvert\Gamma_L\rvert}$	$\frac{1 + \lvert\Gamma_L\rvert}{1 - \lvert\Gamma_L\rvert}$

the input impedance of a terminated lossy transmission line is

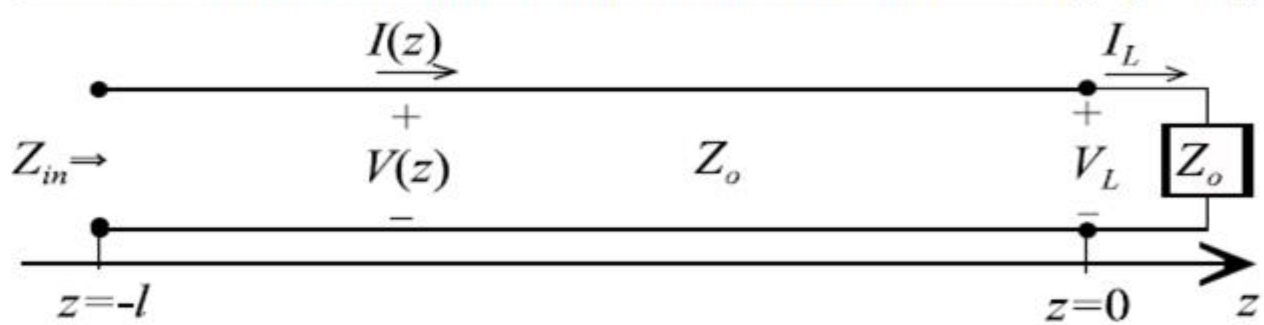


$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(-j\gamma l)}{Z_o + jZ_L \tan(-j\gamma l)} = Z_o \frac{Z_L + Z_o \tanh(\gamma l)}{Z_o + Z_L \tanh(\gamma l)}$$

The impedance at the input of a transmission line of length l terminated with an impedance Z_L is

$$Z_{in} = Z(-l) = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}$$

<u>Lossless Transmission Line with Matched Load</u> $(Z_L = Z_o)$



the general impedance at any point along the length of the transmission line may be written as

$$Z(z) = Z_o \frac{e^{-j\beta z} + \Gamma e^{j\beta z}}{e^{-j\beta z} - \Gamma e^{j\beta z}} = Z_o \frac{1 + \Gamma e^{j2\beta z}}{1 - \Gamma e^{j2\beta z}} = Z_o \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

The normalized value of the impedance $z_n(z)$ is

$$z_n(z) = \frac{Z(z)}{Z_o} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)} = r(z) + jx(z) \tag{3}$$

Example $Z_L = 60 + j50$ $Z_o = 50$ $l = \frac{4}{10}\lambda$

(b.) $$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(60 + j50) - 50}{(60 + j50) + 50} = \frac{10 + j50}{110 + j50} = 0.422\angle 54^o$$

(a.) $$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.422}{1 - 0.422} = 2.46$$

(c.) $$Y_L = \frac{1}{Z_L} = \frac{1}{60 + j50} = (9.84 - j8.2)\ \text{mS}$$

(d.) $$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l} \qquad \beta l = \frac{2\pi}{\lambda}\frac{4}{10}\lambda = \frac{4\pi}{5}$$

$$= 50\,\frac{(60 + j50) + j50\tan\left(\frac{4\pi}{5}\right)}{50 + j(60 + j50)\tan\left(\frac{4\pi}{5}\right)} = (24.5 + j20.3)\ \Omega$$

(e.) $|V(z)| = |V_o^+| \, |1 + \Gamma e^{j2\beta z}|$ $\qquad \Gamma e^{j2\beta z} = |\Gamma| e^{j\theta} e^{j2\beta z}$

$|V(z)|_{\max} = |V_o^+| \, [1 + |\Gamma|]$ $\qquad \theta + 2\beta z_{\max} = n\pi \quad$ (even n)

$|V(z)|_{\min} = |V_o^+| \, [1 - |\Gamma|]$ $\qquad \theta + 2\beta z_{\min} = n\pi \quad$ (odd n)

$$z_{\min} = \frac{n\pi - \theta}{2\beta} = \frac{n\pi - \left(\frac{54\pi}{180}\right)}{4\pi}\lambda = \frac{n - 0.3}{4}\lambda \qquad \text{(odd } n\text{)}$$

$$n = 1 \quad \Rightarrow \quad z_{\min} = \frac{0.7}{4}\lambda = 0.175\lambda$$

$$n = -1 \quad \Rightarrow \quad z_{\min} = -\frac{1.3}{4}\lambda = -0.325\lambda \quad \Rightarrow \quad l_{\min} = 0.325\lambda$$

(f.)

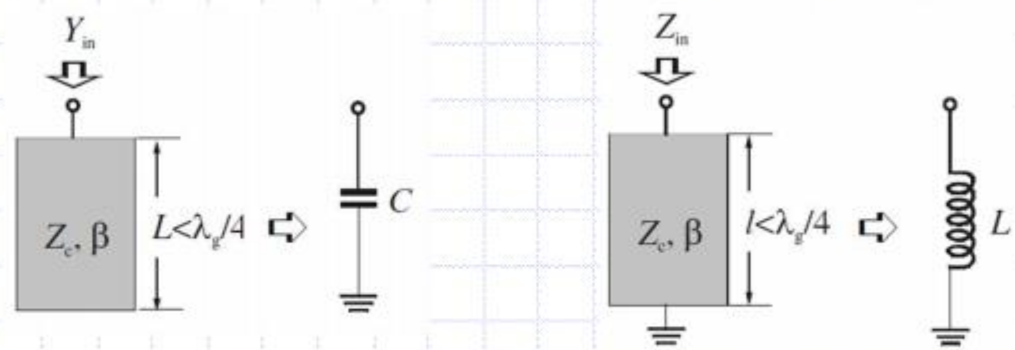
$$z_{\max} = \frac{n\pi - \theta}{2\beta} = \frac{n\pi - \left(\frac{54\pi}{180}\right)}{4\pi}\lambda = \frac{n - 0.3}{4}\lambda \qquad \text{(even } n\text{)}$$

$$n = 0 \quad \Rightarrow \quad z_{\max} = -\frac{0.3}{4}\lambda = -0.075\lambda \quad \Rightarrow \quad l_{\max} = 0.075\lambda$$

$$n = 2 \quad \Rightarrow \quad z_{\max} = \frac{1.7}{4}\lambda = 0.425\lambda$$

➢ Quasilumped elements

- Open- and short-circuited stubs
  (assuming the length L is smaller than a quarter of guided wavelength λ_g)



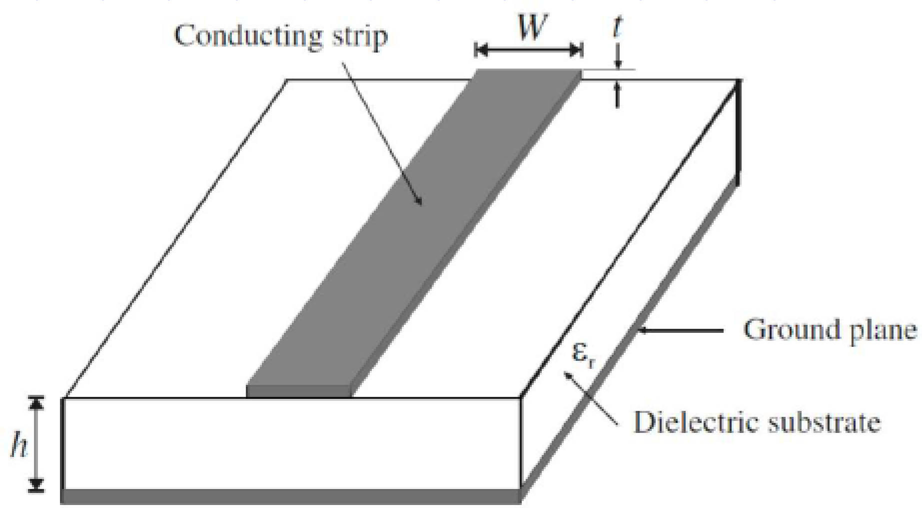
$$Y_{in} = jY_c \tan\left(\frac{2\pi}{\lambda_g} l\right) \underset{\uparrow \; l<\frac{\lambda_g}{8}}{\approx} jY_c\left(\frac{2\pi}{\lambda_g} l\right) = j\omega \overset{C}{\boxed{\left(\frac{Y_c l}{v_p}\right)}}$$

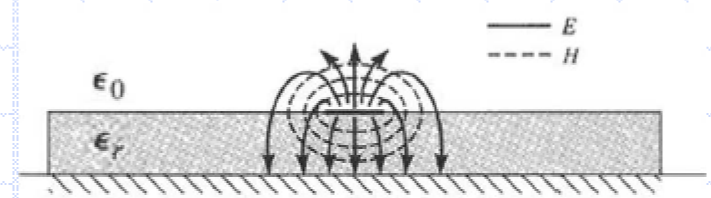
$$Z_{in} = jZ_c \tan\left(\frac{2\pi}{\lambda_g} l\right) \underset{\uparrow \; l<\frac{\lambda_g}{8}}{\approx} jZ_c\left(\frac{2\pi}{\lambda_g} l\right) = j\omega \overset{L}{\boxed{\left(\frac{Z_c l}{v_p}\right)}}$$

- When a transmission line is not terminated with a matched load, it leads to losses and reflections. In order to avoid this, a λ/4 transmission line can be used for matching purpose. The characteristic impedance of the λ/4 transmission line is given by $Z_1=\sqrt{(Z_o R)}L$. substituting the given values, we get $Z_1=70.71\ \Omega$.
- For a transmission line , if the input impedance of the transmission line is 100Ω with a characteristic impedance of 150Ω, then the magnitude of the reflection co efficient. The expression for reflection co-efficient of a transmission line in terms input and characteristic impedance is $(Z_{in}-Z_o)/(Z_{in}+ Z_o)$. Substituting the given values in the above expression, reflection co-efficient is 0.2.

Microstrip Lines

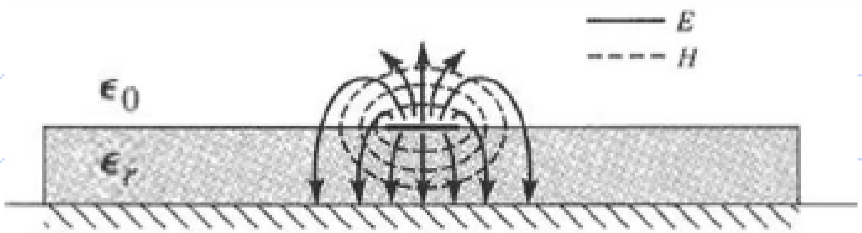
Microstrip Lines

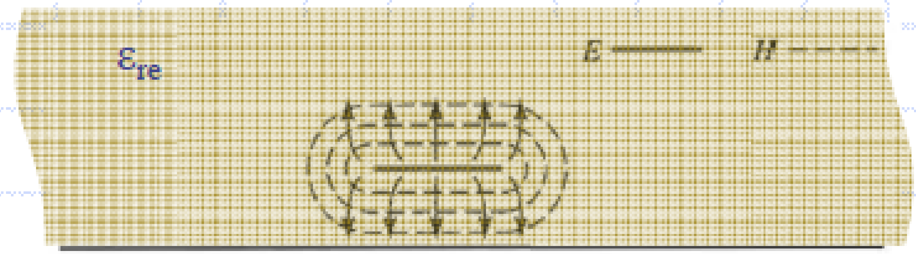




- Transmission Line Parameters

Effective Dielectric Constant (ε_{re}) and Characteristic Impedance(Z_C)





- For thin conductors (i.e., t → 0), closed-form expression (error ≤ 1 %):
  - W/h ≤ 1:

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left\{\left(1 + 12\frac{h}{W}\right)^{-0.5} + 0.04\left(1 - \frac{W}{h}\right)^2\right\}$$

$$Z_c = \frac{\eta}{2\pi\sqrt{\varepsilon_{re}}}\ln\left(\frac{8h}{W} + 0.25\frac{W}{h}\right)$$

  - W/h ≥ 1:

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{W}\right)^{-0.5}$$

$$Z_c = \frac{\eta}{\sqrt{\varepsilon_{re}}}\left\{\frac{W}{h} + 1.393 + 0.677\ln\left(\frac{W}{h} + 1.444\right)\right\}^{-1}$$

For thin conductors (i.e., t → 0), more accurate expressions:

- Effective dielectric constant (error $\leq 0.2\,\%$):

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + \frac{10}{u}\right)^{-ab}$$

$$a = 1 + \frac{1}{49}\ln\left(\frac{u^4 + \left(\frac{u}{52}\right)^2}{u^4 + 0.432}\right) + \frac{1}{18.7}\ln\left[1 + \left(\frac{u}{18.1}\right)^3\right]$$

$$b = 0.564\left(\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right)^{0.053}$$

- Characteristic impedance (error $\leq 0.03\,\%$):

$$Z_c = \frac{\eta}{2\pi\sqrt{\varepsilon_{re}}}\ln\left[\frac{F}{u} + \sqrt{1 + \left(\frac{2}{u}\right)^2}\right]$$

$$F = 6 + (2\pi - 6)\exp\left[-\left(\frac{30.666}{u}\right)^{0.7528}\right]$$

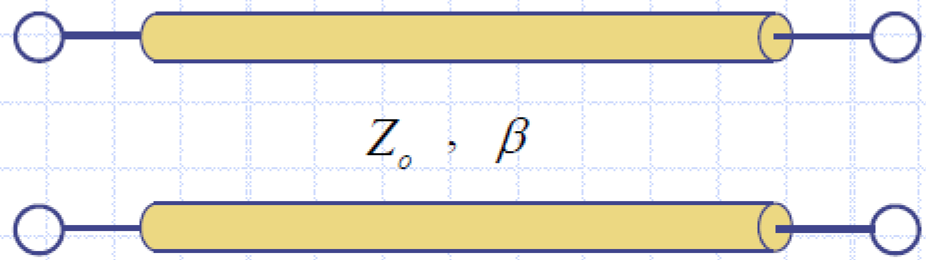
- Transmission Line Parameters

- Guided wavelength

$$\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{re}}} \qquad \text{or} \qquad \lambda_g = \frac{300}{f(GHz)\sqrt{\varepsilon_{re}}}\, mm$$

- Propagation constant

$$\beta = \frac{2\pi}{\lambda_g}$$



- Phase velocity

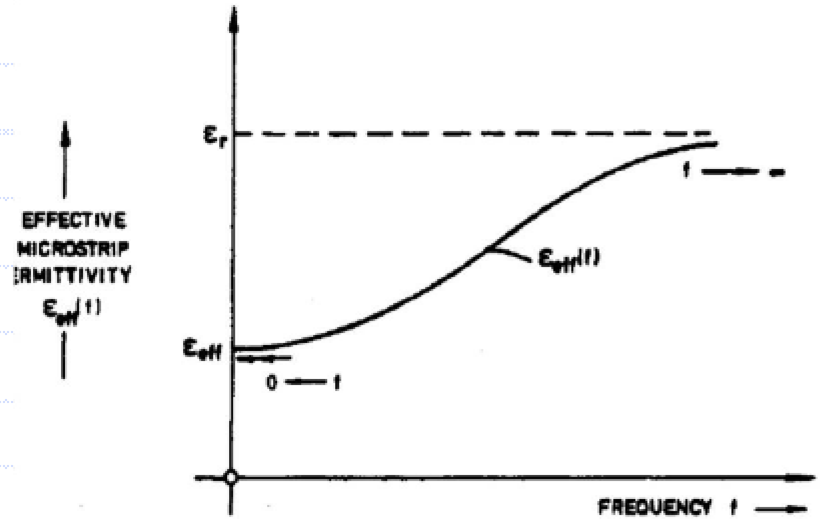
$$\upsilon_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{\varepsilon_{re}}}$$

- Electrical length

$$\theta = \beta \ell$$

- Transmission Line Parameters

- Losses
  - Conductor loss
  - Dielectric loss
  - Radiation loss
- Dispersion
  - $\varepsilon_{re}(f)$
  - $Z_o(f)$



- Surface Waves and higher-order modes
  - Coupling between the quasi-TEM mode and surface wave mode become significant when the frequency is above f_s

$$f_s = \frac{c \tan^{-1} \varepsilon_r}{\sqrt{2}\pi h \sqrt{\varepsilon_r - 1}}$$

  - Cutoff frequency f_c of first higher-order modes in a microstrip

$$f_c = \frac{c}{\sqrt{\varepsilon_r}\,(2W + 0.8h)}$$

  - The operating frequency of a microstrip line < Min (f_s, f_c)

Smith Chart

Smith Chart History

- Invented by Phillip H. Smith in 1939
- Used to solve a variety of transmission line and waveguide problems

Basic Uses

For evaluating the rectangular components, or the magnitude and phase of an input impedance or admittance, voltage, current, and related transmission functions at all points along a transmission line, including:

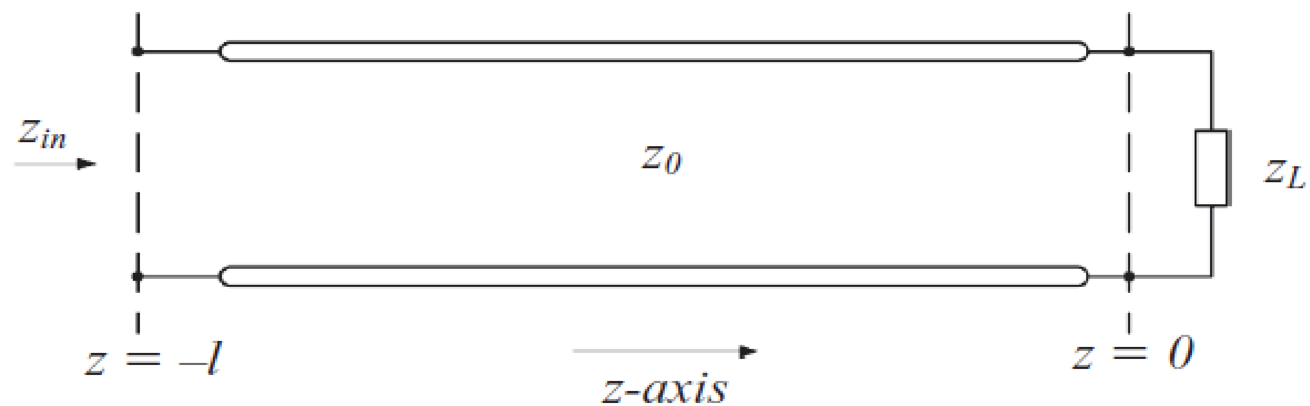
- Complex voltage and current reflections coefficients
- Complex voltage and current transmission coefficents
- Power reflection and transmission coefficients
- Reflection Loss
- Return Loss
- Standing Wave Loss Factor
- Maximum and minimum of voltage and current, and SWR
- Shape, position, and phase distribution along voltage and current standing waves

SPECIAL CASE OF LOSSLESS TRANSMISSION LINES

TABLE 2.2 Formulas for Transmission Lines

Quantity	General Line	Lossless Line
Propagation constant, $\gamma = \alpha + j\beta$	$\sqrt{(R + j\omega L)(G + j\omega C)}$	$j\omega\sqrt{LC}$
Phase constant, β	$\mathrm{Im}(\gamma)$	$\omega\sqrt{LC} = \frac{w}{v} = \frac{2\pi}{\lambda}$
Attenuation constant, α	$\mathrm{Re}(\gamma)$	0
Characteristic impedance, Z_0	$\sqrt{\frac{R + j\omega L}{G + j\omega C}}$	$\sqrt{\frac{L}{C}}$
Input impedance, Z_{in}	$Z_0 \frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l}$	$Z_0 \frac{Z_L \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jZ_L \sin \beta l}$
Impedance of shorted line	$Z_0 \tanh \gamma l$	$jZ_0 \tan \beta l$
Impedance of open line	$Z_0 \coth \gamma l$	$-jZ_0 \cot \beta l$
Impedance of quarter-wave line	$Z_0 \frac{Z_L \sinh \alpha l + Z_0 \cosh \alpha l}{Z_0 \sinh \alpha l + Z_L \cosh \alpha l}$	$\frac{Z_0^2}{Z_L}$
Impedance of half-wave line	$Z_0 \frac{Z_L \cosh \alpha l + Z_0 \sinh \alpha l}{Z_0 \cosh \alpha l + Z_L \sinh \alpha l}$	Z_L
Reflection coefficient, Γ_L	$\frac{Z_L - Z_0}{Z_L + Z_0}$	$\frac{Z_L - Z_0}{Z_L + Z_0}$
Voltage standing-wave ratio (VSWR)	$\frac{1 + \lvert\Gamma_L\rvert}{1 - \lvert\Gamma_L\rvert}$	$\frac{1 + \lvert\Gamma_L\rvert}{1 - \lvert\Gamma_L\rvert}$

Terminated Transmission Line



- ***Input impedance*** *and* ***reflection coefficient***

$$Z_{in}(z) = \frac{V(z)}{I(z)} = Z_0 \frac{A_1 e^{-\gamma z} + A_2 e^{\gamma z}}{A_1 e^{-\gamma z} - A_2 e^{\gamma z}} = Z_0 \frac{e^{-\gamma z} + \Gamma_0 e^{\gamma z}}{e^{-\gamma z} - \Gamma_0 e^{\gamma z}}$$

where $\Gamma_0 = A_2 / A_1$ is called the *reflection coefficient* at the load

$$\boxed{\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\Gamma(z) = \frac{V_-(z)}{V_+(z)} = \frac{A_2 e^{\gamma z}}{A_1 e^{-\gamma z}} = \Gamma_0 e^{2\gamma z} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{2\gamma z}$$

Note: the power reflection coefficient is:

$$\Gamma_P(l) = |\Gamma(l)|^2 = |\Gamma_0|^2 e^{-2\alpha l}$$

The input impedance $$Z_{in}(l) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$

For the lossless case $$\boxed{Z_{in}(l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}}$$

Input impedance for special cases

- *Matched case (Γ = 0):*

$$Z_{in}(l) = Z_0$$

- *Open circuit (Γ = 1):*

$$Z_{in}(l) = Z_0 \frac{1}{j\tan(\beta l)}$$

- *Short circuit (Γ = -1):*

$$Z_{in}(l) = jZ_0\tan(\beta l)$$

- *Quarter-wavelength case:*

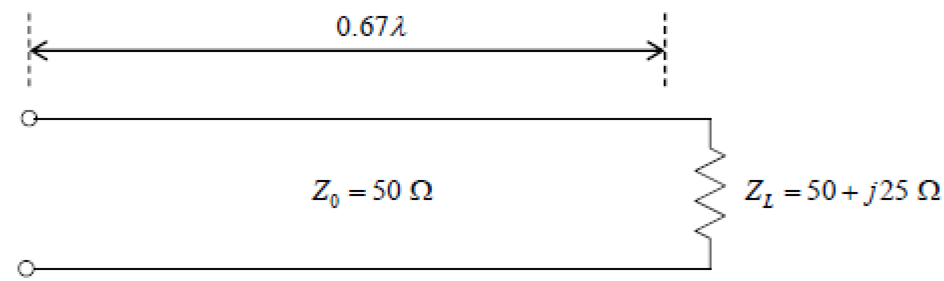
$$Z_{in}(l) = \frac{Z_0^2}{Z_L}$$

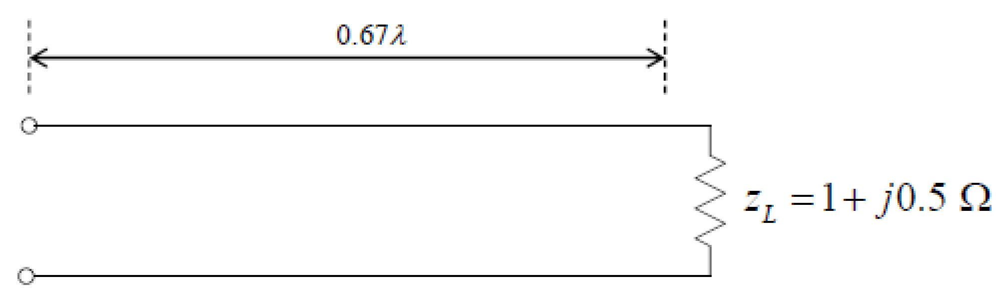
Return loss

- When the voltage reflection coefficient and power reflection coefficient are expressed in logarithmic forms, they give the same result, which is called the ***return loss***

$$L_{RT}(l) = -20\log_{10}(|\Gamma(l)|) = -10\log_{10}(\Gamma_P(l))$$

Example Impedance Trans.
Normalized the parameters





$$z_{\text{in}}(\ell) = \frac{z_L + j\tan\beta\ell}{1 + jz_L\tan\beta\ell} = \frac{(1+j0.5) + j\tan(2\pi\cdot 0.67)}{1 + j(1+j0.5)\tan(2\pi\cdot 0.67)} = 1.299 - j0.485$$

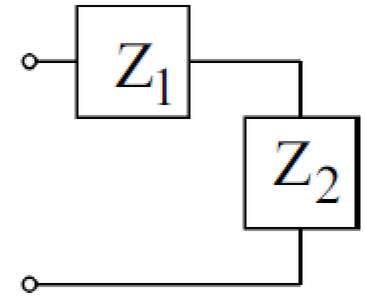
Admittance

A matching network is going to be a combination of elements connected in series AND parallel.

Impedance is well suited when working with series configurations. For example:

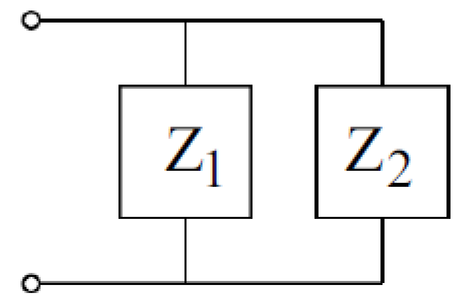
$V = ZI$

$$Z_L = Z_1 + Z_2$$



Impedance is NOT well suited when working with parallel configurations.

$$Z_L = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

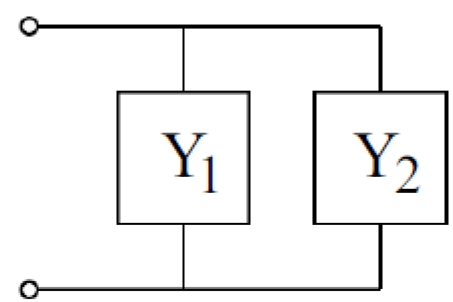


For parallel loads it is better to work with admittance.

$I = YV$

$$Y_1 = \frac{1}{Z_1}$$

$$Y_L = Y_1 + Y_2$$



Normalized Impedance

All impedances are normalized. This is usually done with respect to the characteristic impedance of the transmission line Z_0.

$$z = \frac{Z}{Z_0}$$

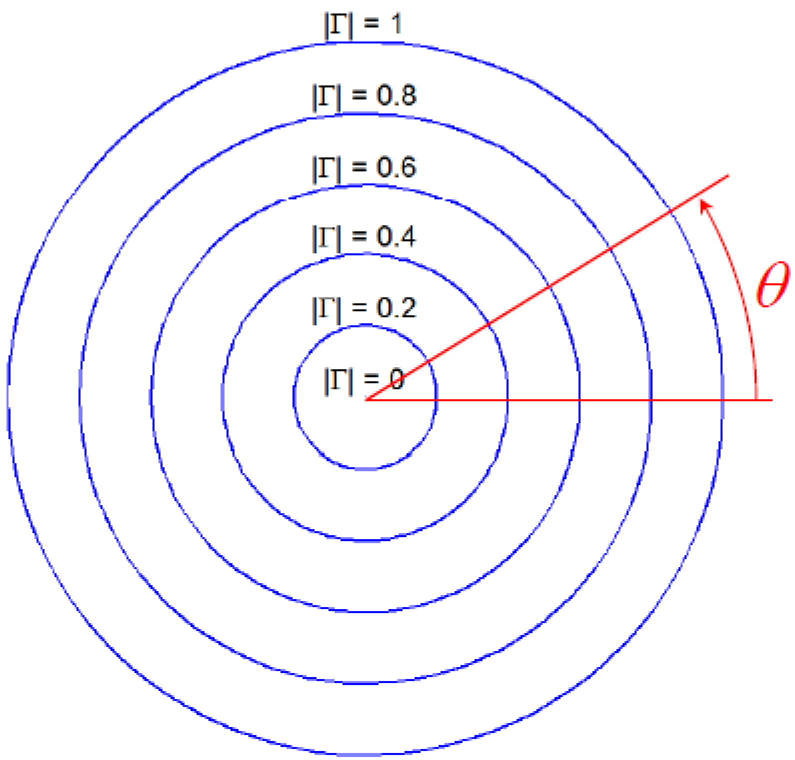
Smith Chart

- Smith chart is based on the polar plot of voltage reflection co-efficient.
- let the reflection co-efficient be expressed in terms of magnitude and direction as $\Gamma = |\Gamma| e^{j\theta}$. Magnitude is plotted as radius from the center of the chart, and the angle is measured in counter clockwise direction from the right hand side. Hence, smith chart is based on the polar pot of voltage reflection co-efficient.
- Reflection co-efficient is defined as the ratio of reflected voltage or current to the incident voltage or current. Hence reflection co-efficient can never be greater than 1. Hence, only reflection co-efficient less than or equal to 1 can be plotted.

Polar Plot of Reflection Coefficient

The Smith chart is based on a polar plot of the voltage reflection coefficient Γ. The outer boundary corresponds to $|\Gamma| = 1$. The reflection coefficient in any passive system must be $|\Gamma| \le 1$.

$$\Gamma = |\Gamma| \cdot e^{j\theta}$$



$|\Gamma| \equiv$ radius on Smith chart

$\theta \equiv$ angle measured CCW from right side of chart

The Smith Chart

$Z = R +/- j X$

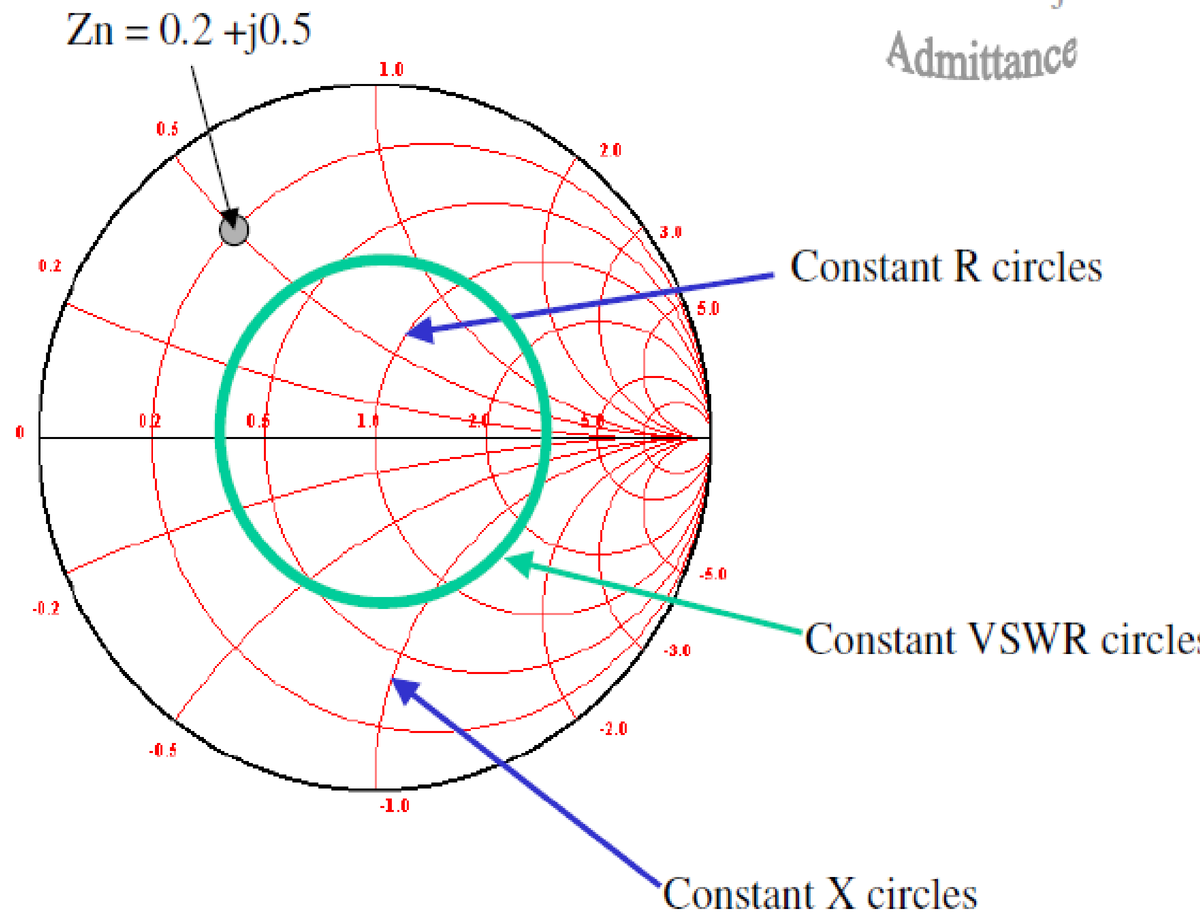
Impedance

$Y = G +/- j B$

Admittance

The chart is normalized (Zn) so that any characteristic impedance (Zo) can be used.

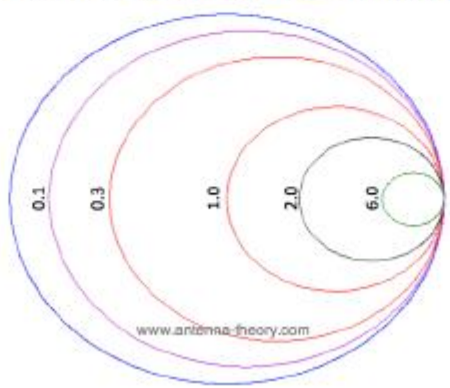
$$Zn = \frac{R}{Zo} +/- j \frac{X}{Zo}$$



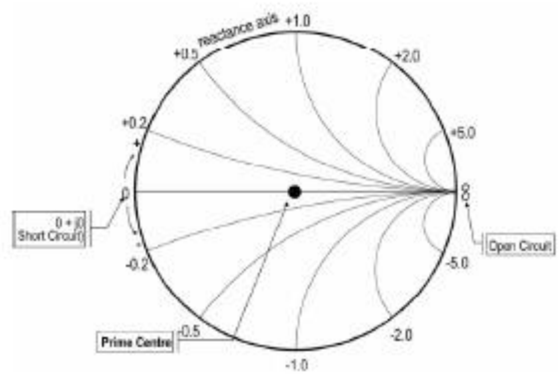
Reflection Coefficient form Normalized Impedance

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} \qquad z_L = \frac{1+\Gamma}{1-\Gamma}$$

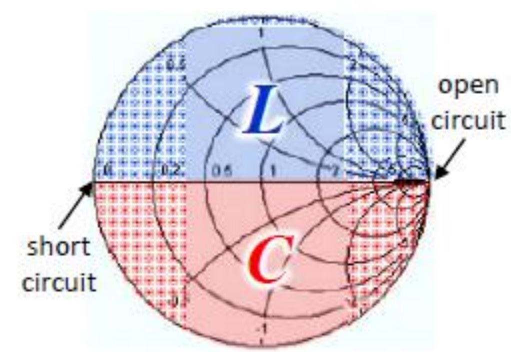
Lines of constant resistance



Lines of constant reactance



Reactance Regions



S: standing-wave ratio

S is numerically equal to the value of r_0 at $P_{\max}$, the point at which the SWR circle intersects the real Γ axis to the right of the chart's center.

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

For $\Gamma_i = 0$

$$\Gamma = \Gamma_r = \frac{r_0 - 1}{r_0 + 1}$$

$$S = \frac{|\widetilde{V}|_{\max}}{|\widetilde{V}|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \text{(dimensionless)}$$

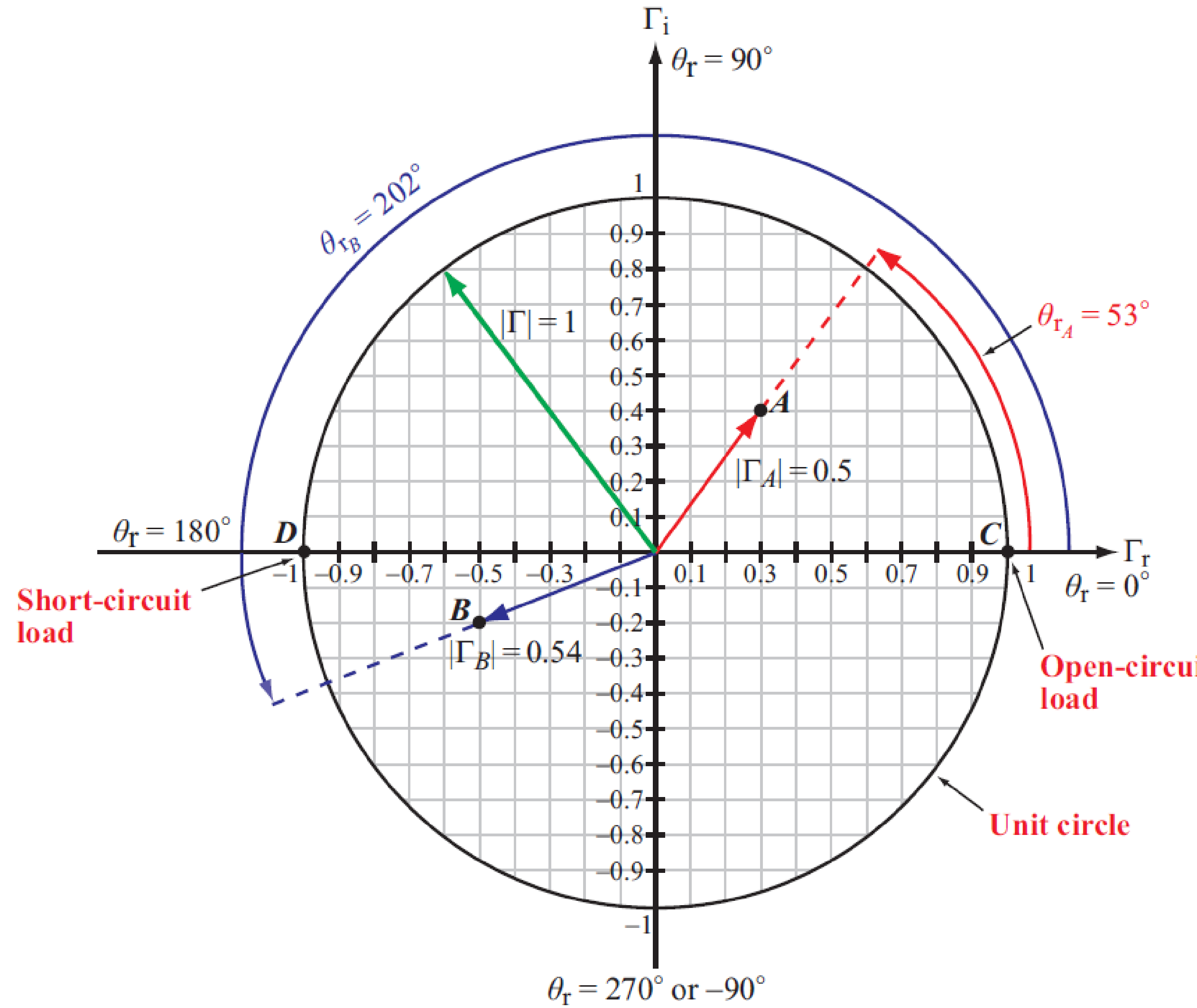
$$|\Gamma| = \frac{S - 1}{S + 1}$$

Complex Plane

$$\Gamma = |\Gamma| e^{j\theta_r} = \Gamma_r + j\Gamma_i$$

$$\Gamma_A = 0.3 + j0.4 = 0.5e^{j53^\circ}$$

$$\Gamma_B = -0.5 - j0.2 = 0.54e^{j202^\circ}$$



Reflection coefficient at the load

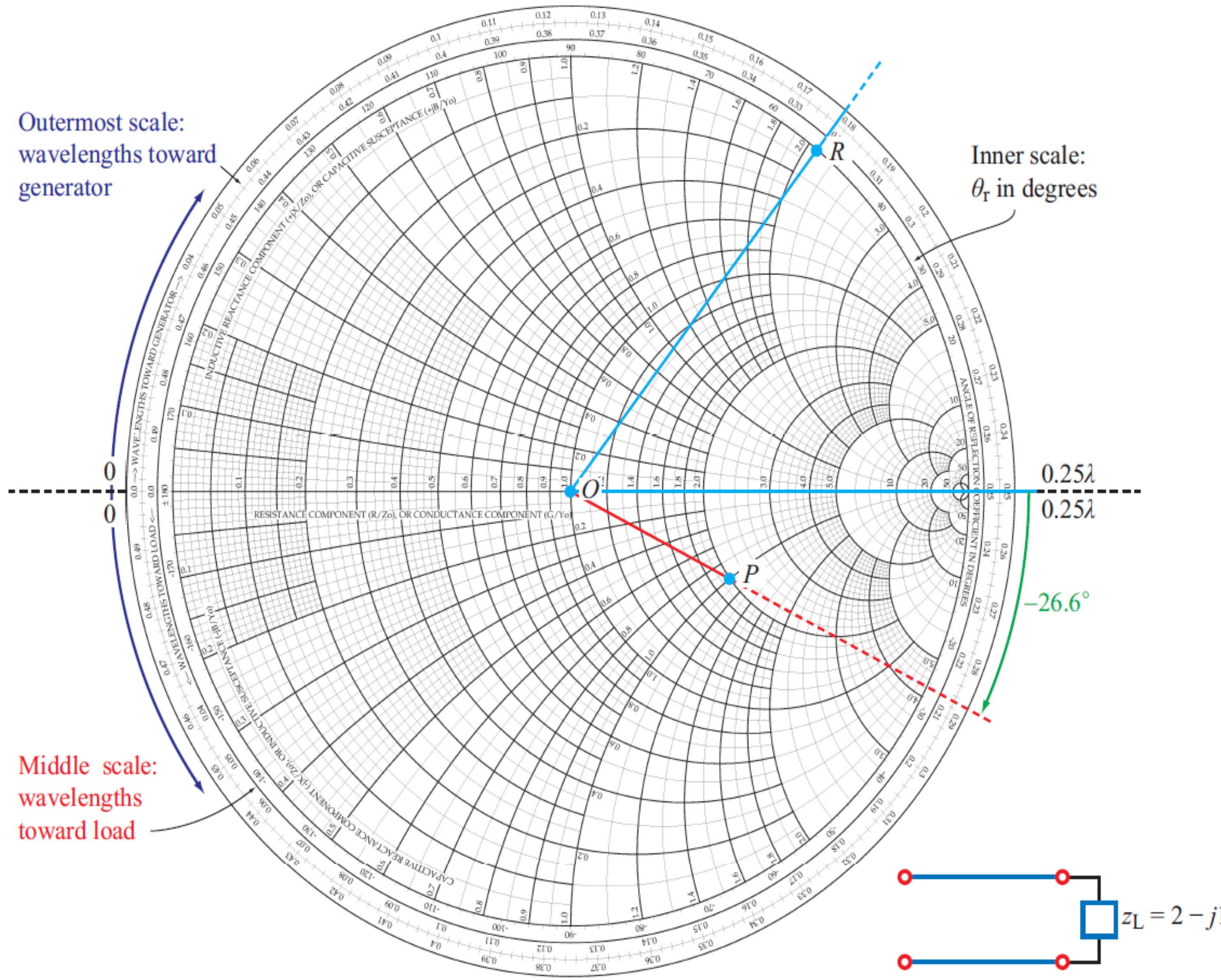


Figure 2-26: Point P represents a normalized load impedance $z_L = 2 - j1$. The reflection coefficient has a magnitude $|\Gamma| = \overline{OP}/\overline{OR} = 0.45$ and an angle $\theta_r = -26.6°$. Point R is an arbitrary point on the $r_L = 0$ circle (which also is the $|\Gamma| = 1$ circle).

Smith Chart

Impedances, voltages, currents, etc. all repeat every half wavelength

$$\Gamma = \Gamma_L = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1+\Gamma}{1-\Gamma} = r_L + jx_L$$

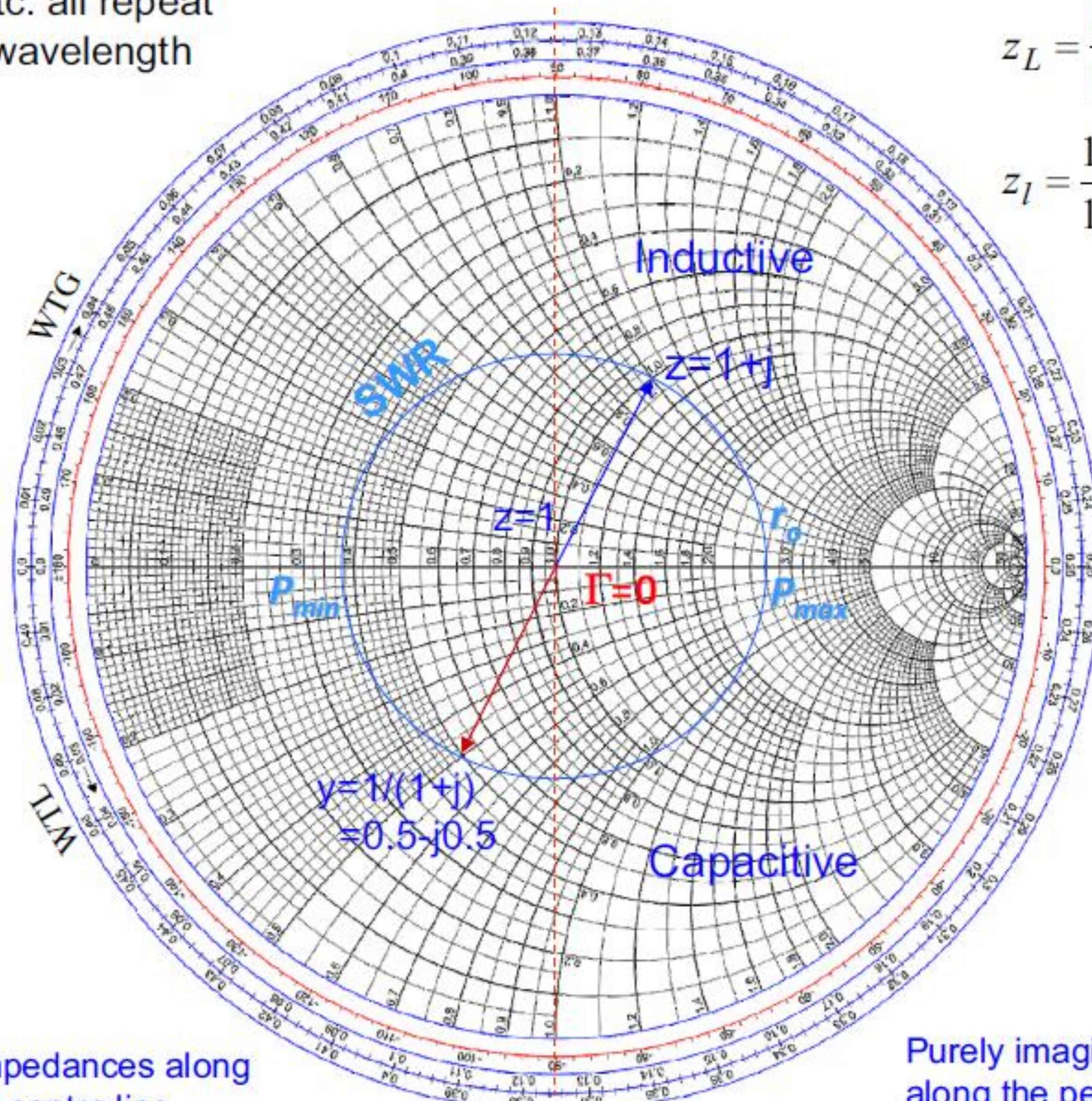
$$z_l = \frac{1+\Gamma_l}{1-\Gamma_l} \qquad \Gamma_l = \Gamma e^{-j2\beta l}$$

$$= \Gamma e^{-j\frac{4\pi}{\lambda}l}$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = r_o \,(\text{at}\, P\,\text{max})$$

Short (z=0)

Γ=-1



Open (z=∞)

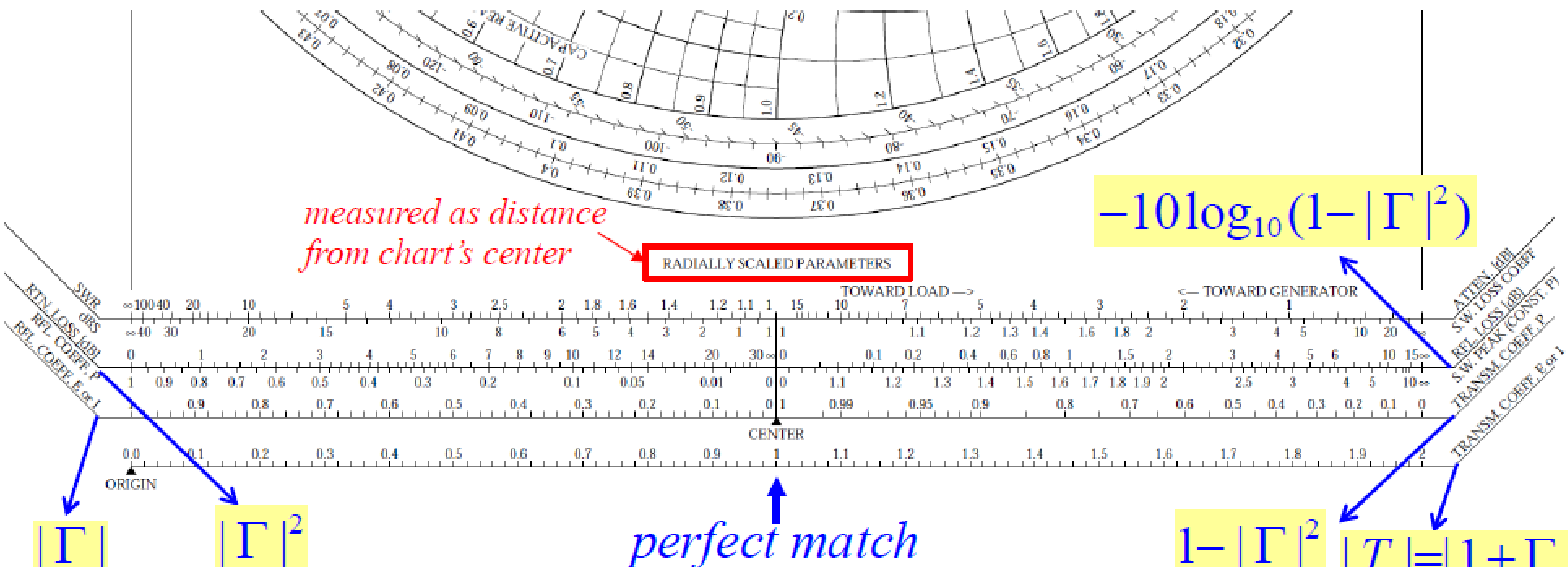
Γ=1

Purely real impedances along the horizontal centre line

Purely imaginary impedances along the periphery

The Smith Chart: Nomographs

- at the bottom of Smith's chart (**left side**), *nomograph* is added to read out with a ruler the following
  - (1st ruler) above: *SWR*, below: *SWR* in dB $(20\log_{10} SWR)$
  - (2nd ruler) above: return loss in dB, $-20\log_{10}|\Gamma|$
    below: power reflection coefficient $|\Gamma|^2$ (P)
  - (3rd ruler) above: reflection coefficient $|\Gamma|$ (E or I)



Example 2-11: Smith Chart Calculations

A 50-Ω lossless transmission line of length 3.3λ is terminated by a load impedance $Z_L = (25 + j50)\ \Omega$.

$$z_L = \frac{Z_L}{Z_0} = \frac{25 + j50}{50} = 0.5 + j1$$

(a)

$$d_{max} = (0.25 - 0.135)\lambda = 0.115\lambda$$

$$d_{min} = (0.5 - 0.135)\lambda = 0.365\lambda$$

(b)

$$z_{in} = 0.28 - j0.40$$

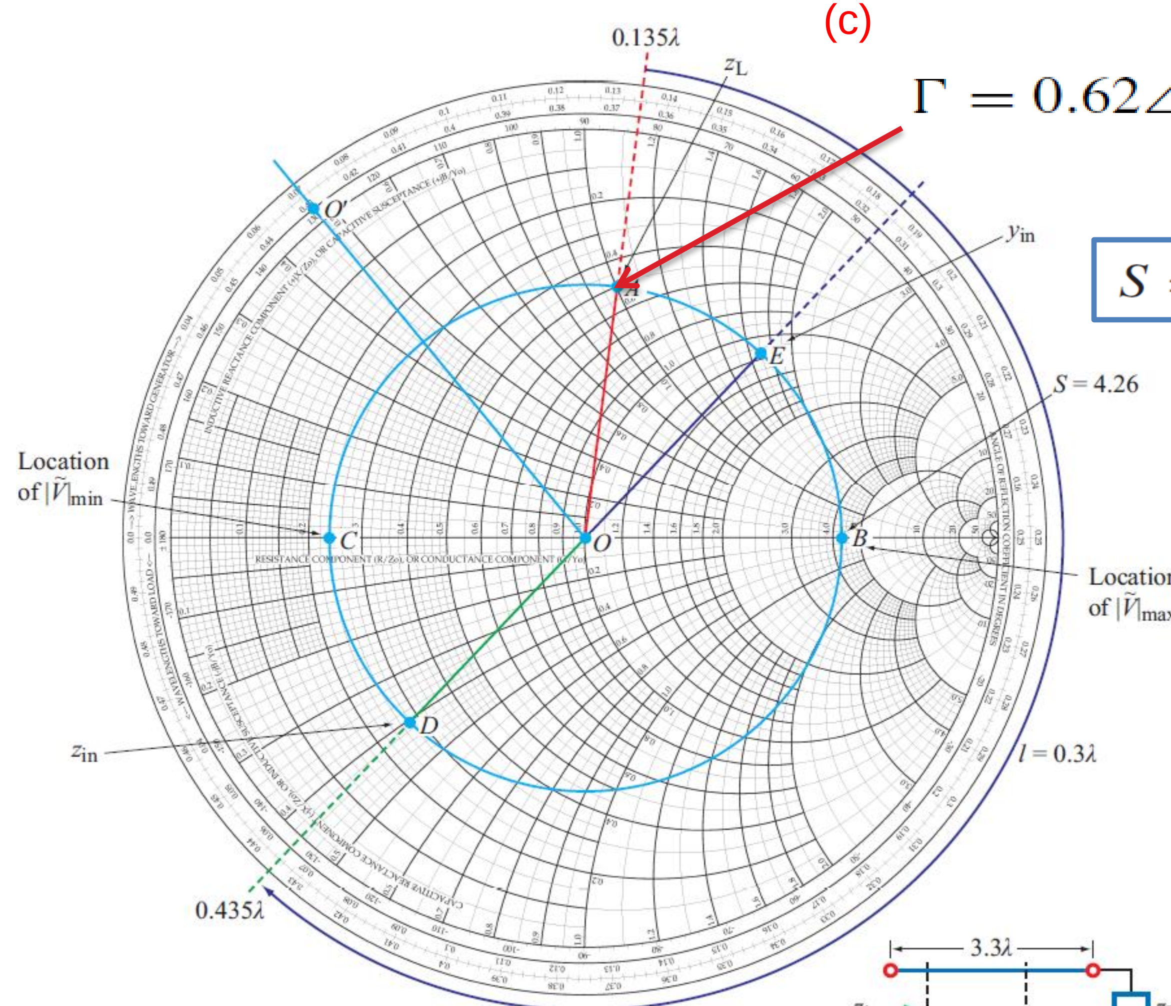
$$Z_{in} = z_{in} Z_0 = (0.28 - j0.40)50 = (14 - j20)\ \Omega$$

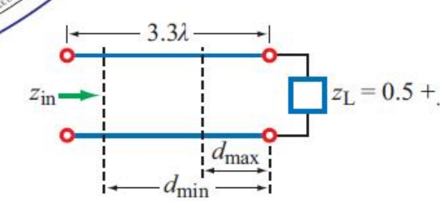
(c)

$$\Gamma = 0.62\angle 83°$$

(d)

$$\boxed{S = 4.26}$$





Example: Consider the characteristic impedance of a 50Ω termination and the following impedances:

$Z_1 = 100 + j50\Omega$ $Z_2 = 75 - j100\Omega$

$Z_5 = \infty$ (an open circuit) $Z_6 = 0$ (a short circuit)

$Z_3 = j200\Omega$ $Z_4 = 150\Omega$

$Z_7 = 50\Omega$ $Z_8 = 184 - j900\Omega$

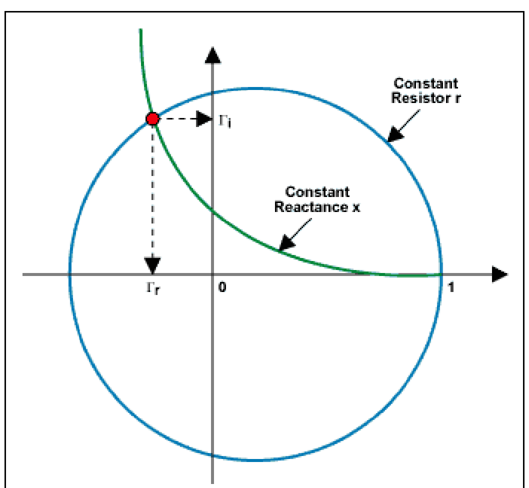
Then, normalize and The points are plotted as follows:

$z_1 = 2 + j$ $z_2 = 1.5 - j2$

$z_5 = 8$ $z_6 = 0$

$z_3 = j4$ $z_4 = 3$

$z_7 = 1$ $z_8 = 3.68 - j18$

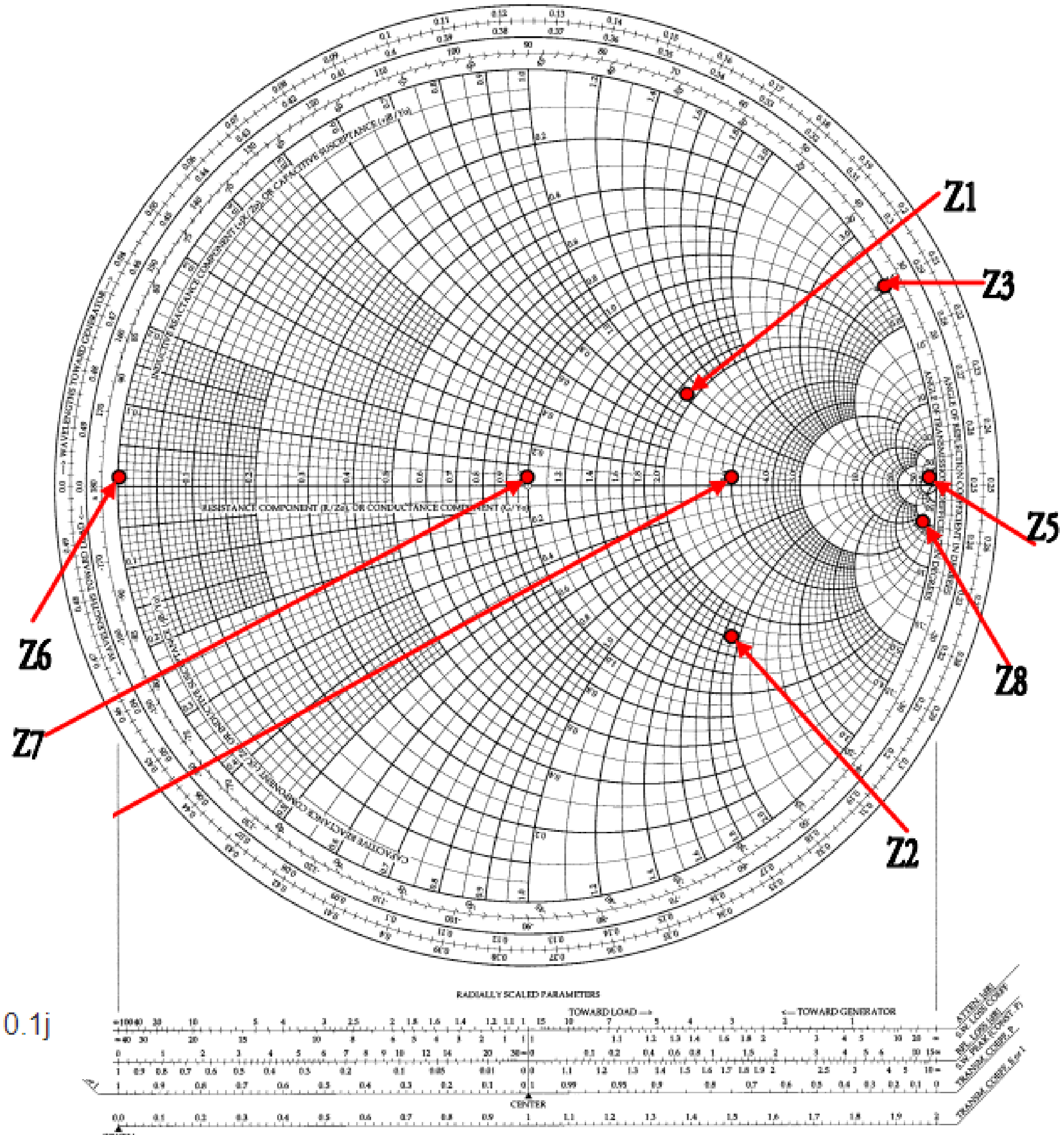


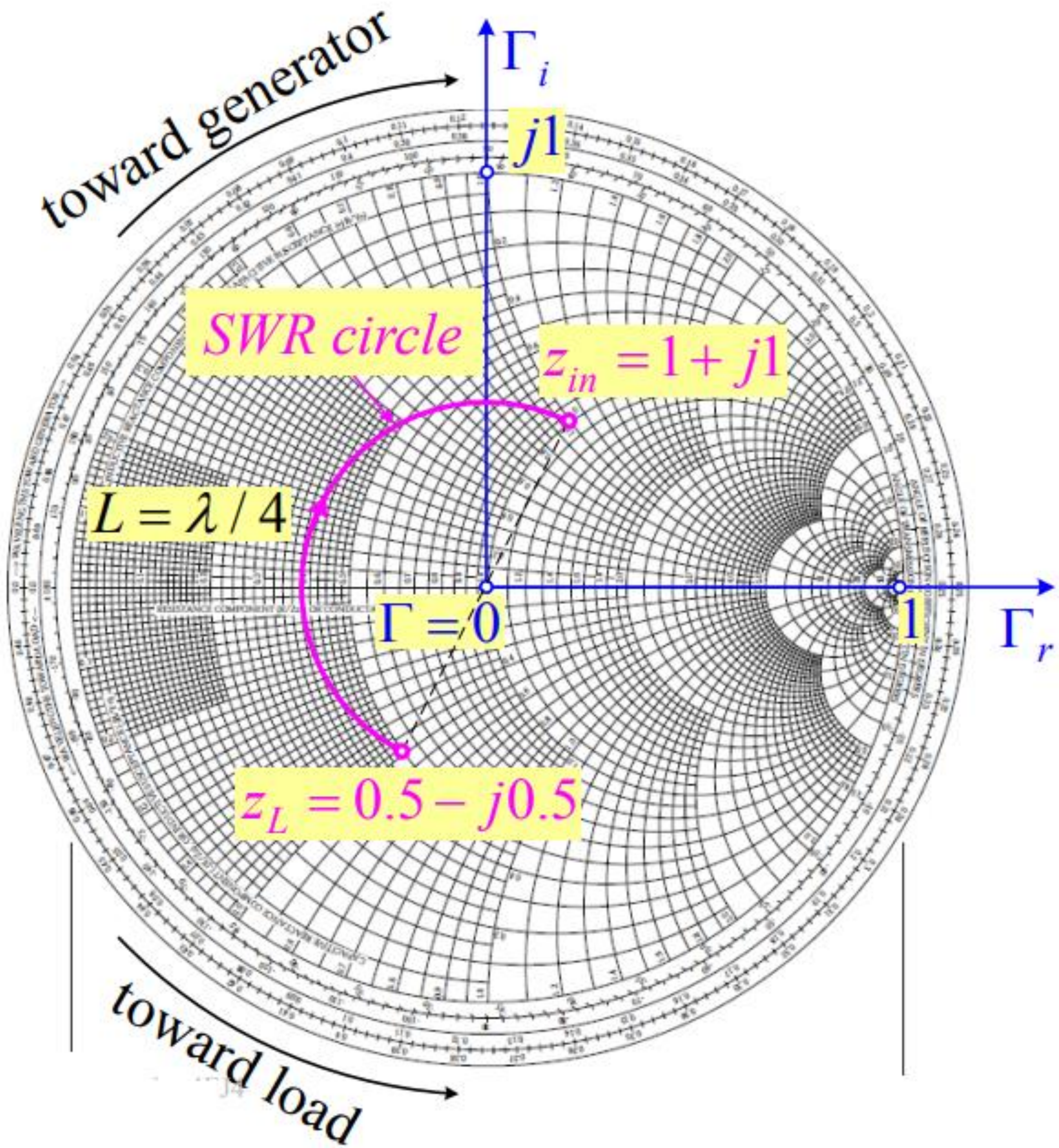
$\Gamma_1 = 0.4 + 0.2j$ $\Gamma_2 = 0.51 - 0.4j$

$\Gamma_5 = 1$ $\Gamma_6 = -1$

$\Gamma_3 = 0.875 + 0.48j$ $\Gamma_4 = 0.5$

$\Gamma_7 = 0$ $\Gamma_8 = 0.96 - 0.1j$





for $Z_0 = 50\ \Omega$, the quarter-wavelength TL transforms a load of

$$Z_L = 25 - j25\ \Omega$$

to an input impedance of

$$Z_{in} = 50 + j50\ \Omega$$

- unknown distance to load in terms of λ

  $D_n = D / \lambda$

- known load Z_L

  $Z_L = 75 + j75\ \Omega$

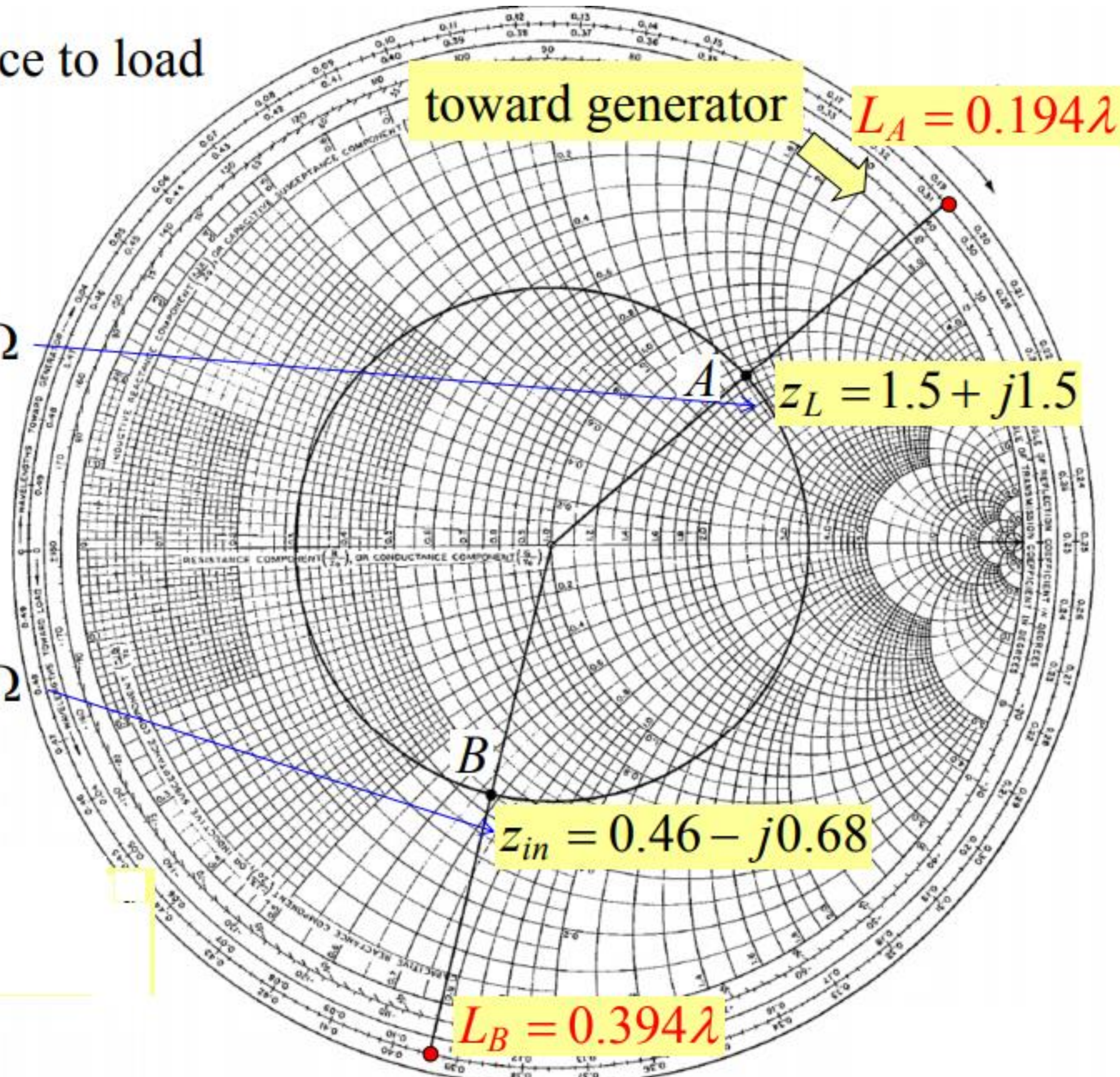
- known Z_0

  $Z_0 = 50\ \Omega$

- measured Z_{in}

  $Z_{in} = 23 - j34\ \Omega$

New position =
$.2\ \lambda + .194\ \lambda =$
$.394\ \lambda$



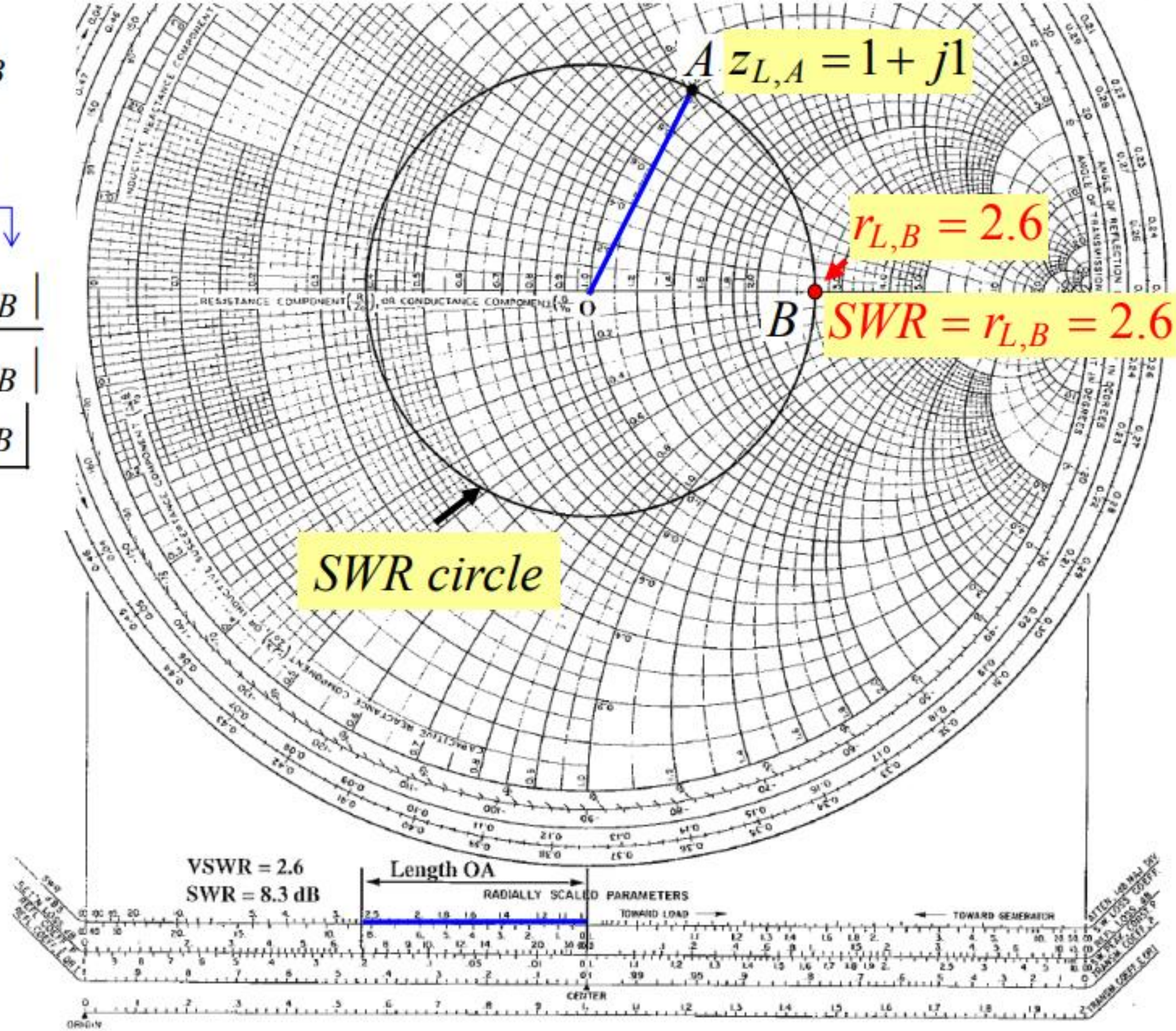
The Smith Chart: Reading Out SWR

$$SWR_A = SWR_B$$

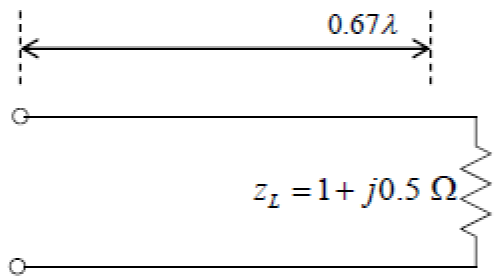
$$\Gamma_B = \frac{r_{L,B} - 1}{r_{L,B} + 1}$$

$$SWR_B = \frac{1 + |\Gamma_B|}{1 - |\Gamma_B|}$$

$$\Rightarrow \boxed{SWR_B = r_{L,B}}$$



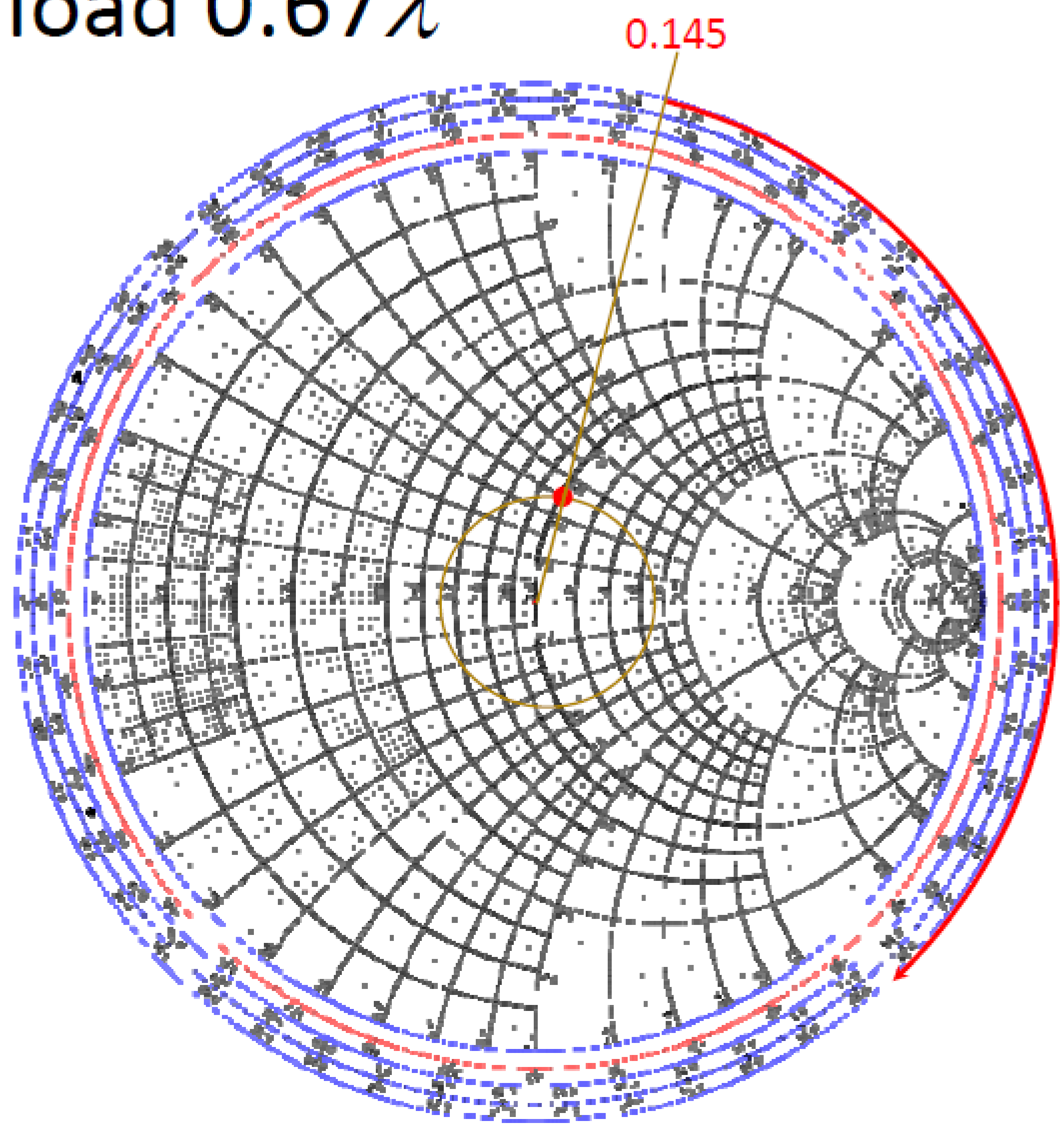
Example Impedance Trans.
Walk away from load 0.67λ



Since the Smith chart repeats every 0.5λ, traversing 0.67λ is the same as traversing 0.17λ.

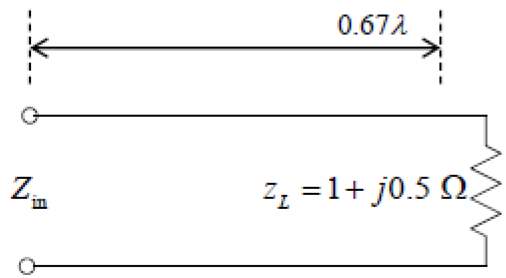
Here we start at 0.145 on the Smith chart.

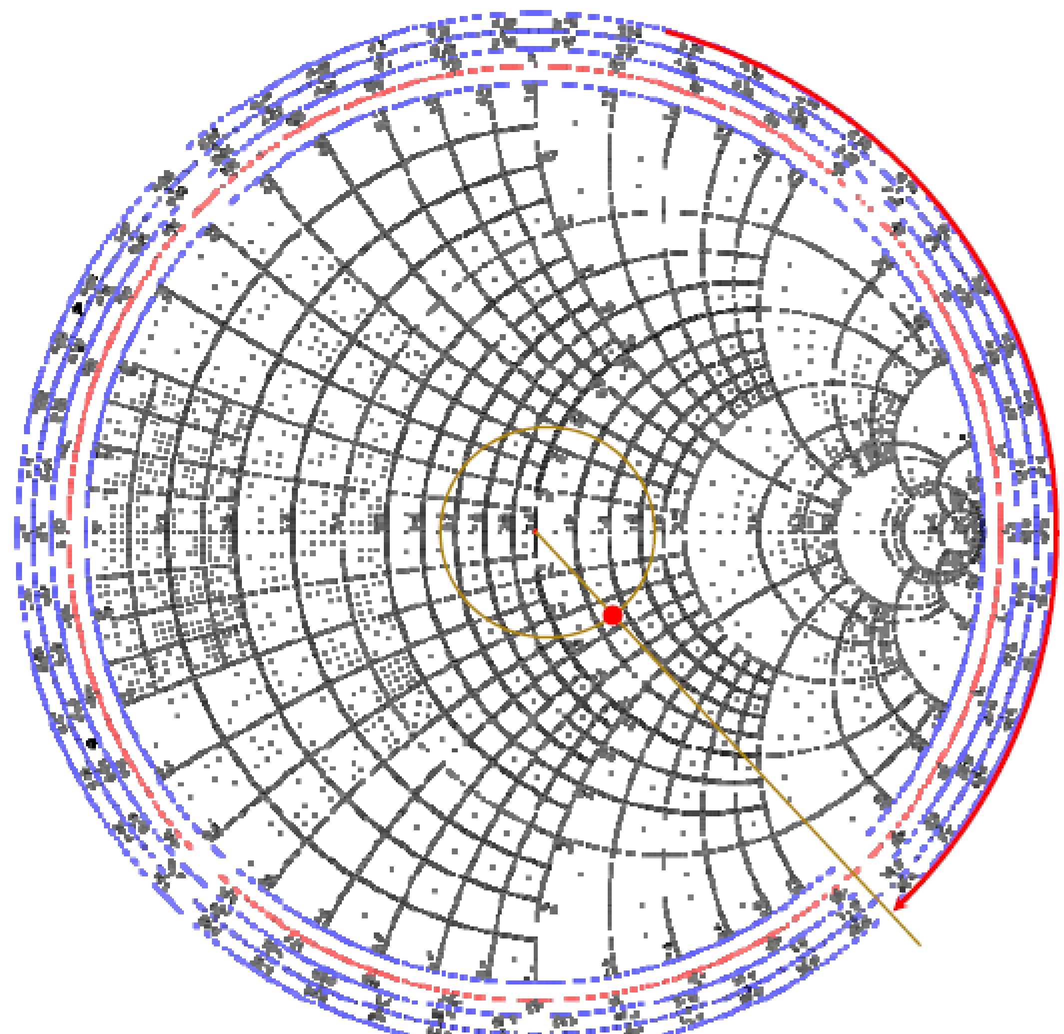
We traverse around the chart to
0.145 + 0.17 = 0.315.



Example Impedance Trans.

Determine input impedance





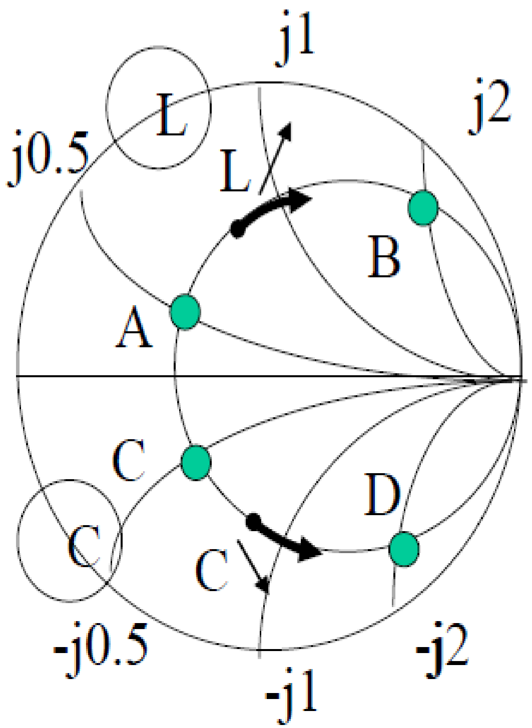
Reflection at the load will be the same regardless of the length of line.

Therefore the VSWR will the same.

The input impedance must lie on the same VSWR plane.

$z_{in} \approx 1.3 - j0.5$

$$Z_{in} = Z_0 z_{in} \approx (50\ \Omega)(1.3 - j0.5) = \boxed{65 - j25\ \Omega}$$



constant R-circle → L or C in series

(1) $A \to B: 1 + j0.5 + jx = 1 + j2 \to jX = j1.5Z_o = j\omega L \to L = \dfrac{1.5Z_o}{\omega}$

: add an L in series

(2) $B \to A: 1 + j2 + jx = 1 + j0.5 \to jX = -j1.5Z_o = \dfrac{1}{j\omega C} \to C = \dfrac{1}{1.5\omega Z_o}$

: add a C in series

or $B \to A: 1 + j2 - j\Delta x = 1 + j0.5 \to j\Delta X = j1.5Z_o = j\omega\Delta L \to \Delta L = \dfrac{1.5Z_o}{\omega}$

: reduce extra $j\Delta X = j1.5Z_o$ (or reduce series L by $\Delta L = \dfrac{1.5Z_o}{\omega}$)

(3) $C \to D: 1 - j0.5 + jx = 1 - j2 \to jX = -j1.5Z_o = \dfrac{1}{j\omega C} \to C = \dfrac{1}{1.5\omega Z_o}$

: add a C in series

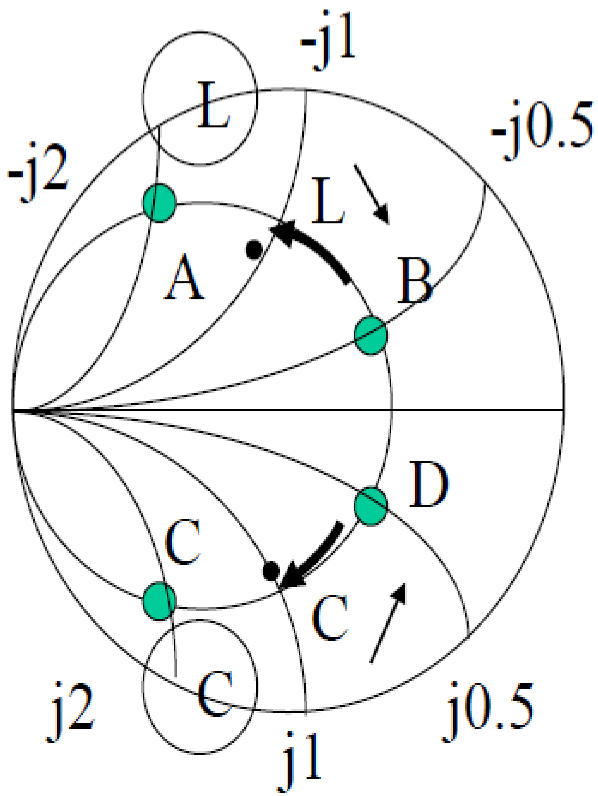
(4) $D \to C: 1 - j2 + jx = 1 - j0.5 \to jX = j1.5Z_o = j\omega L \to L = \dfrac{1.5Z_o}{\omega}$

: add an L in series

or $D \to C: 1 - j2 - j\Delta x = 1 - j0.5 \to j\Delta X = -j1.5Z_o = \dfrac{1}{j\omega\Delta C} \to \Delta C = \dfrac{1}{1.5\omega Z_o}$

reduce extra $j\Delta X = -j1.5Z_o$ (or reduce series C by $\Delta C = \dfrac{1}{1.5\omega Z_o}$)

constant G-circle → L or C in shunt



(1) $A \rightarrow B: 1-j2+jb=1-j0.5 \rightarrow jB=j1.5Y_o=j\omega C \rightarrow C=\dfrac{1.5Y_o}{\omega}$

: add a C in shunt

or $A \rightarrow B: 1-j2-j\Delta b=1-j0.5 \rightarrow j\Delta B=-j1.5Y_o=\dfrac{1}{j\omega\Delta L} \rightarrow \Delta L=\dfrac{1}{1.5\omega Y_o}$

: reduce shunt $j\Delta B=-j1.5Y_o$(or reduce shunt L by $\Delta L=\dfrac{1}{1.5\omega Y_o}$)

(2) $B \rightarrow A: 1-j0.5+jb=1-j2 \rightarrow jb=-j1.5=\dfrac{1}{j\omega L}$

: add an L in shunt

(3) $C \rightarrow D: 1+j2+jb=1+j0.5 \rightarrow jb=-j1.5=\dfrac{1}{j\omega L}$

: add an L in shunt

or $C \rightarrow D: 1+j2-j\Delta b=1+j0.5 \rightarrow j\Delta B=j1.5Y_o=j\omega\Delta C \rightarrow \Delta C=\dfrac{1.5Y_o}{\omega}$

reduce shunt $j\Delta B=j1.5Y_o$(or reduce shunt C by $\Delta C=\dfrac{1.5Y_o}{\omega}$)

(4) $D \rightarrow C: 1+j0.5+jb=1+j2 \rightarrow jb=j1.5=j\omega C$

: add a C in shunt

Impedance/Admittance Conversion

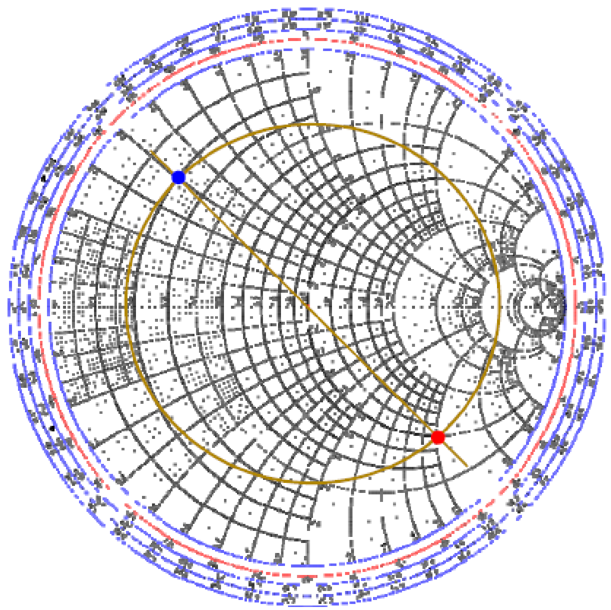
The Smith chart is just a plot of complex numbers. These could be admittance as well as impedance.

To determine admittance from impedance (or the other way around)…

1. Plot the impedance point on the Smith chart.
2. Draw a circle centered on the Smith chart that passes through the point (i.e. constant VSWR).
3. Draw a line from the impedance point, through the center, and to the other side of the circle.
4. The intersection at the other side is the admittance.

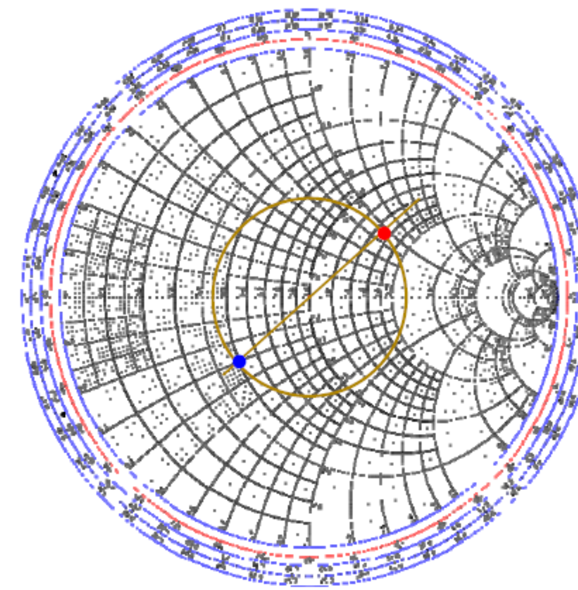
$z = 0.2 + j0.4$

$\boxed{y = 1.0 - j2.0}$



$z = 0.5 - j0.3$

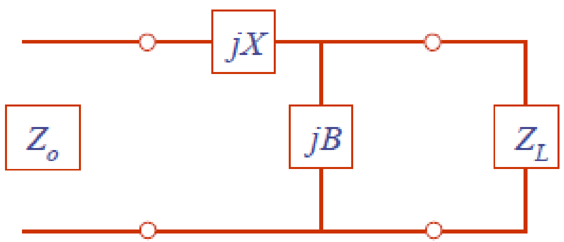
$\boxed{y = 1.5 + j0.9}$

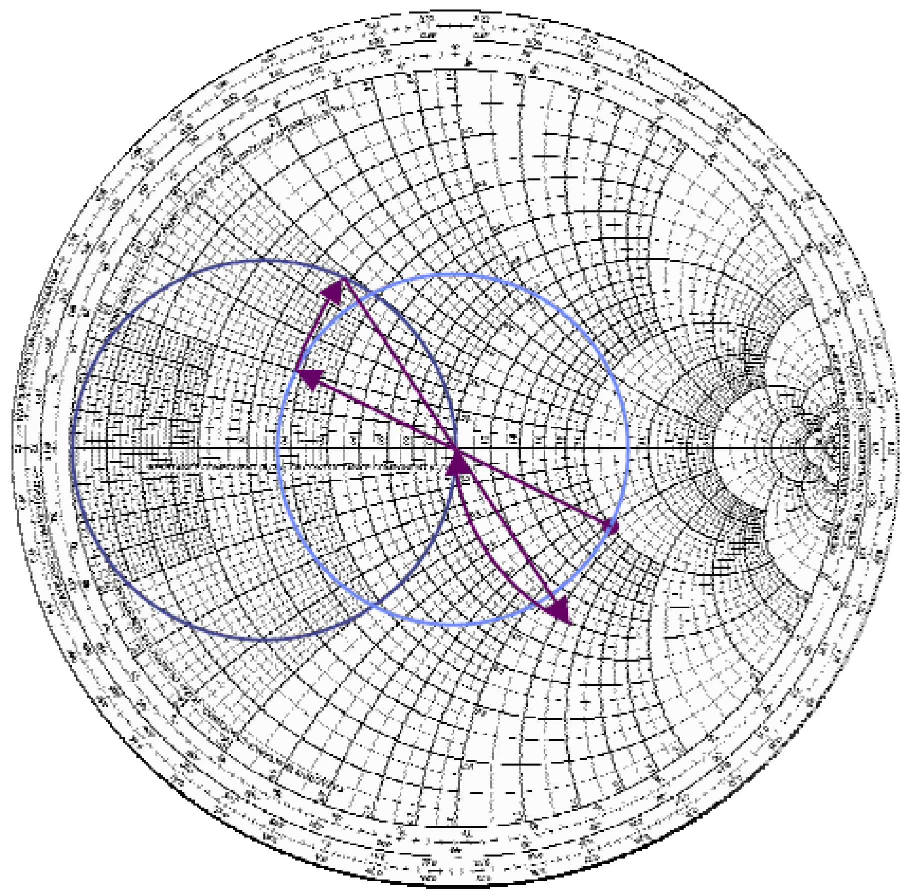


Smith Chart Solution

Therefore we have $b = 0.3$, $x = 1.2$ (check this result with the analytic solution). Then for a frequency at $f = 500$ MHz, we have

$$C = \frac{b}{2\pi f Z_0} = 0.92\,pF \qquad L = \frac{xZ_0}{2\pi f} = 38.8\,nH$$

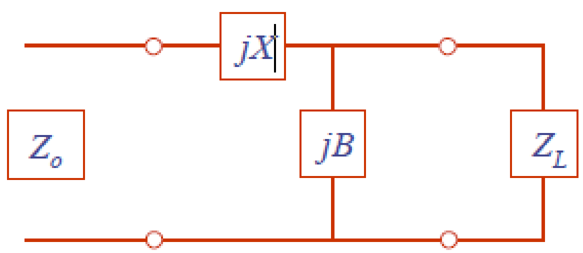


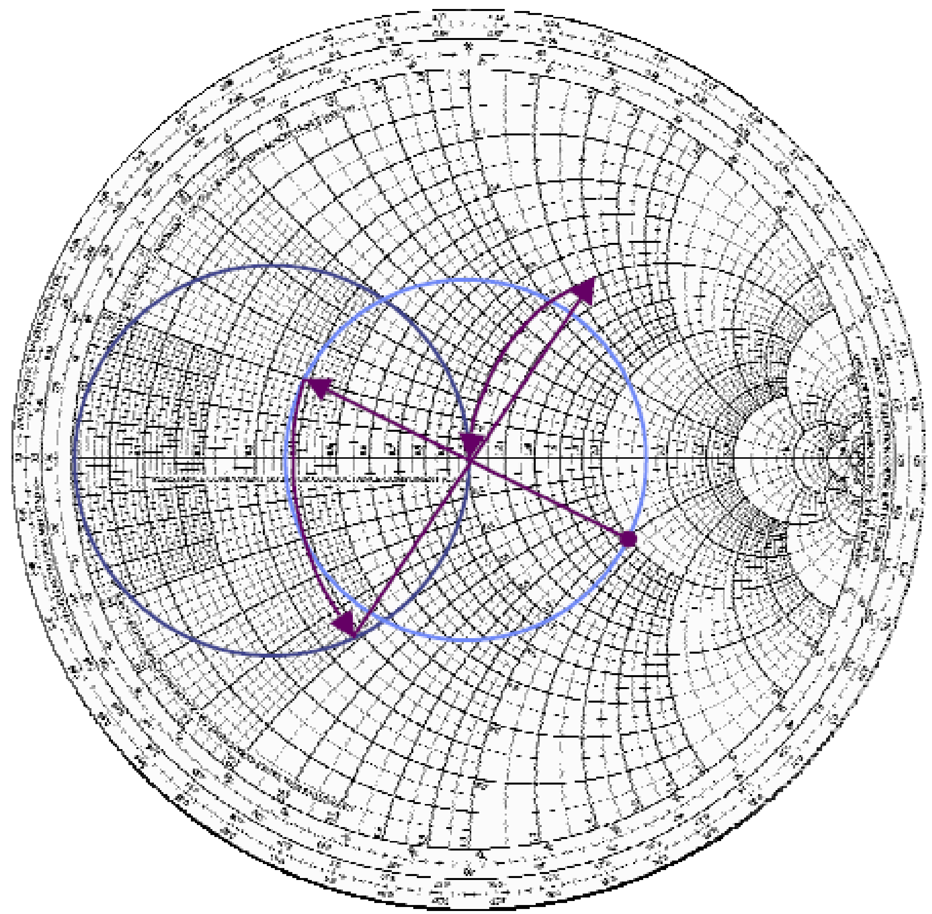


- Z_L=200–j100Ω, Z_o=100Ω, f_o=500MHz.
- Plot z_L =2–j1
- Draw SWR and y =1 circles
- Convert z_L to y_L
- Add shunt susceptance to y_L
- Convert y to z
- Add series reactance

D.M. Pozar, *Microwave Engineering*, 2nd Edition, Example 5.1, p.p. 254, John Wiley & Sons, 1998

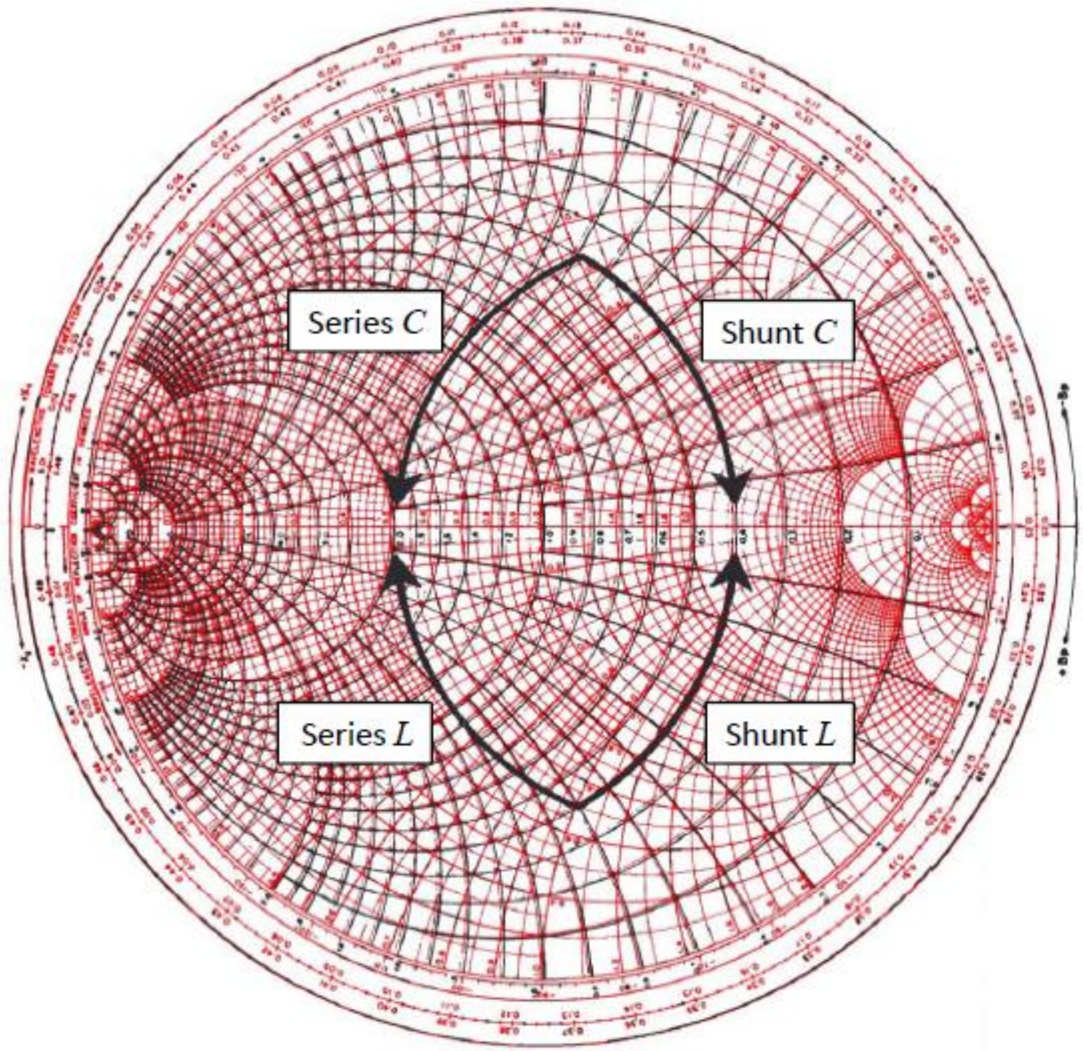
Smith Chart Solution

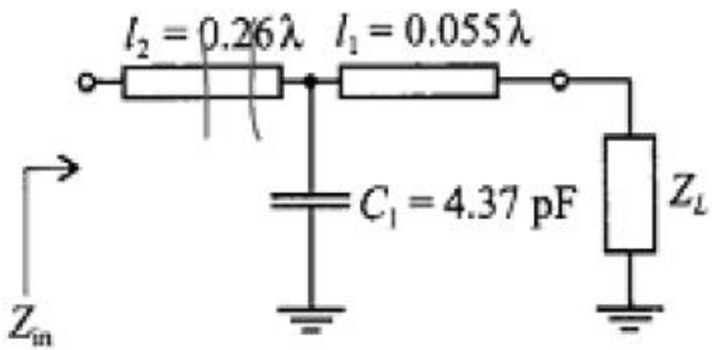




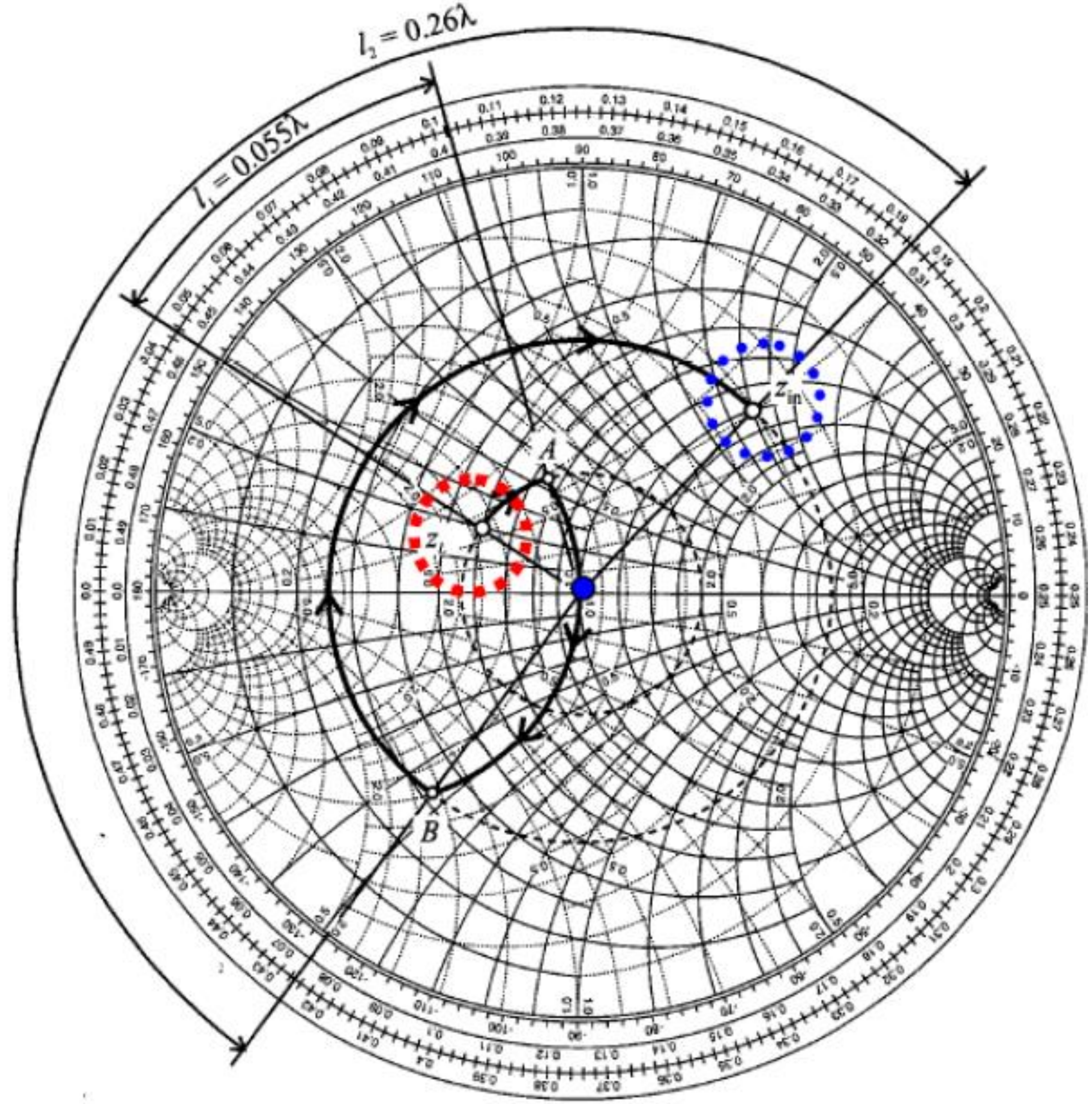
- Z_L=200–j100Ω, Z_o=100Ω, f_o=500MHz.
- Plot z_L =2–j1
- Draw SWR and y =1 circles
- Convert z_L to y_L
- Add shunt susceptance to y_L
- Convert y to z
- Add series reactance

D.M. Pozar, *Microwave Engineering*, 2nd Edition, Example 5.1, p.p. 254, John Wiley & Sons, 1998


Series C
Shunt C
Series L
Shunt L



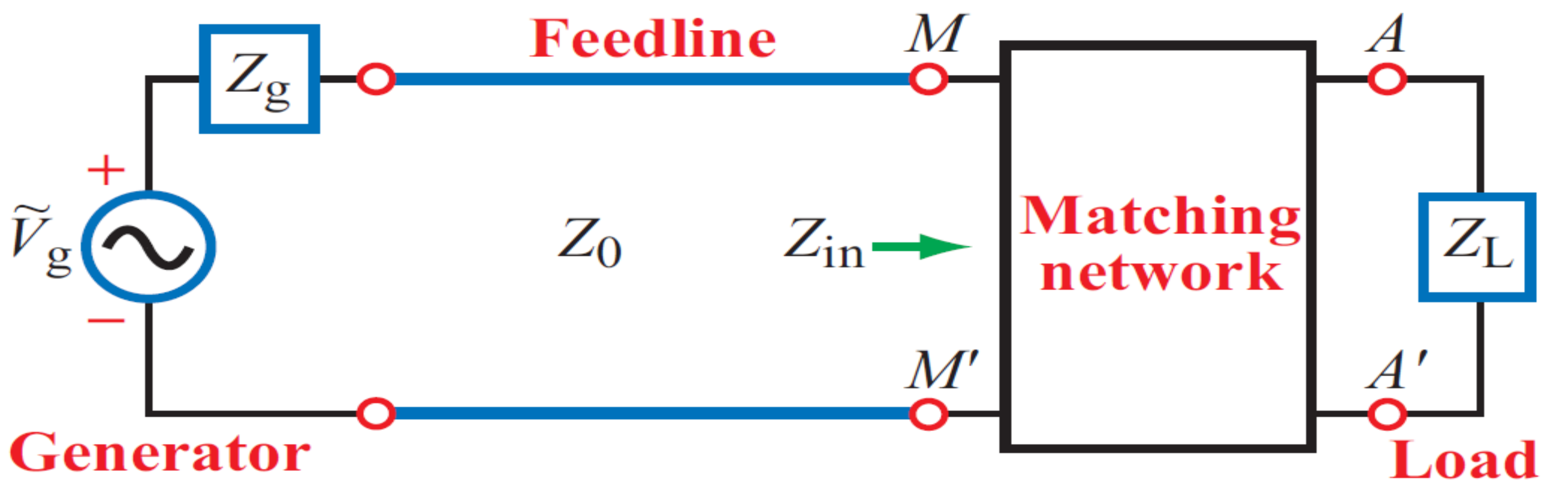
$l_2 = 0.26\lambda$
$l_1 = 0.055\lambda$
$C_1 = 4.37$ pF
Z_L
Z_{in}



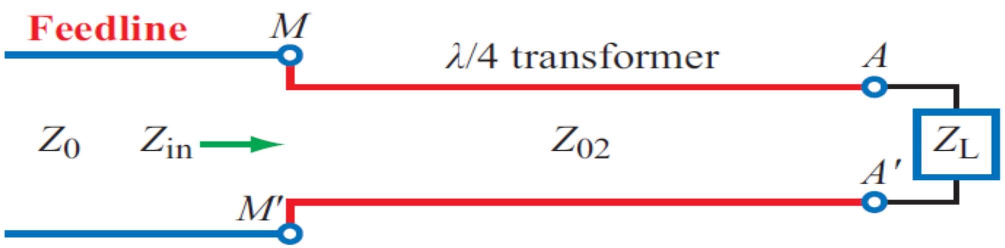
$l_2 = 0.26\lambda$
$l_1 = 0.055\lambda$
z_{in}
A
z_L
B

Matching Networks

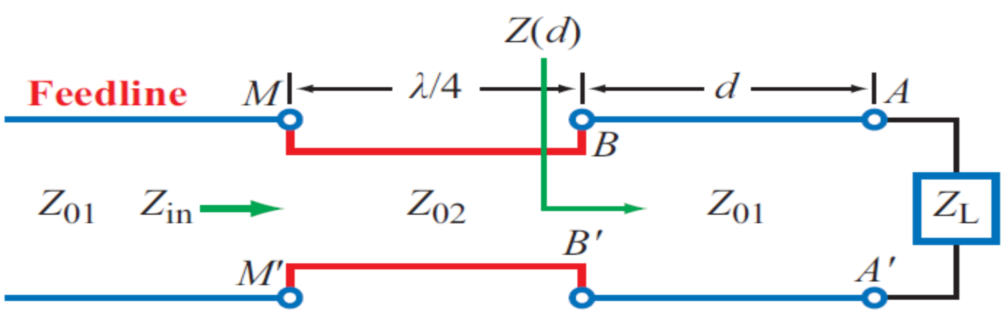
The purpose of the matching network is to eliminate reflections at terminals MM' for waves incident from the source. Even though multiple reflections may occur between AA' and MM', only a forward traveling wave exists on the feedline.



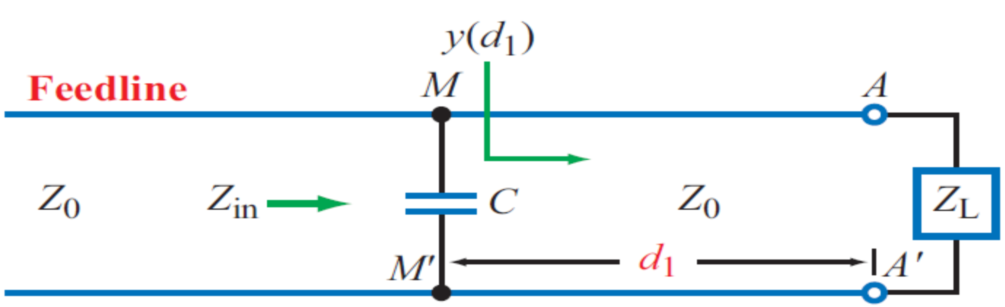
Examples of Matching Networks



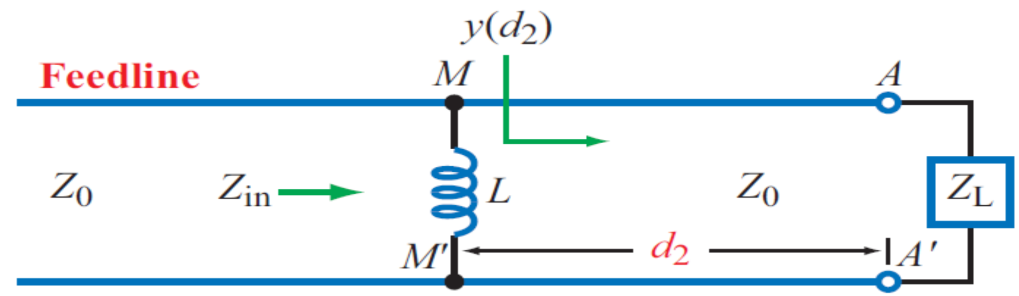
(a) In-series $\lambda/4$ transformer inserted at AA'



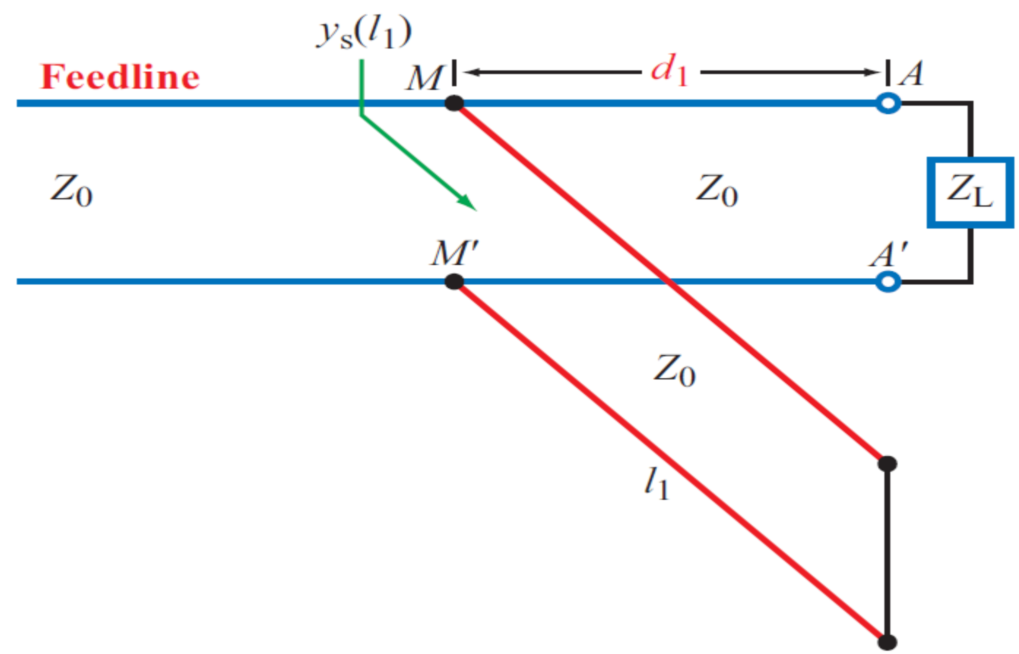
(b) In-series $\lambda/4$ transformer inserted at $d = d_{max}$ or $d = d_{min}$



(c) In-parallel insertion of capacitor at distance d_1



(d) In-parallel insertion of inductor at distance d_2



(e) In-parallel insertion of a short-circuited stub

Lumped-Element Matching

Choose d and Ys to achieve a match at MM'

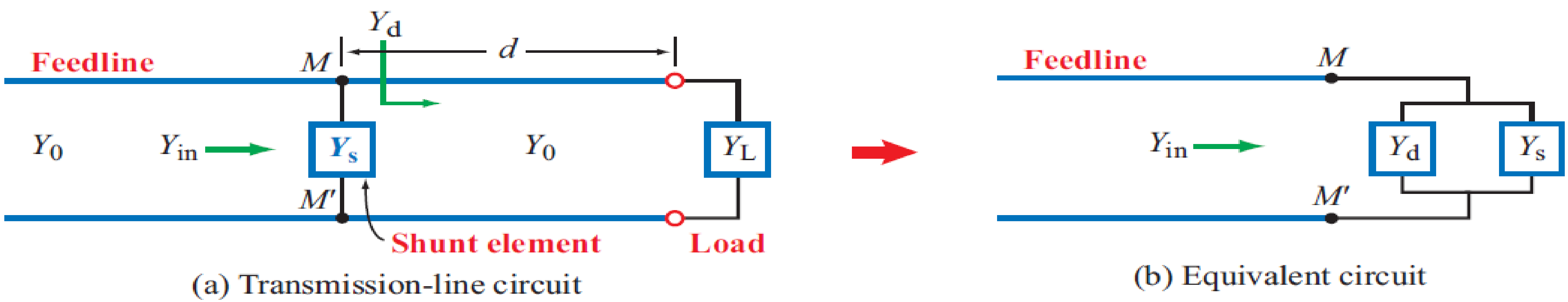


Figure 2-34: Inserting a reactive element with admittance Y_s at MM' modifies Y_d to Y_{in}.

$$Y_{in} = Y_d + Y_s$$

$$Y_{in} = (G_d + jB_d) + jB_s$$
$$= G_d + j(B_d + B_s).$$

$$y_{in} = g_d + j(b_d + b_s). \tag{2.140}$$

To achieve a matched condition at MM', it is necessary that $y_{in} = 1 + j0$, which translates into two specific conditions, namely

$$g_d = 1 \quad \text{(real-part condition)}, \tag{2.141a}$$
$$b_s = -b_d \quad \text{(imaginary-part condition)}. \tag{2.141b}$$

Smith Chart Example 1

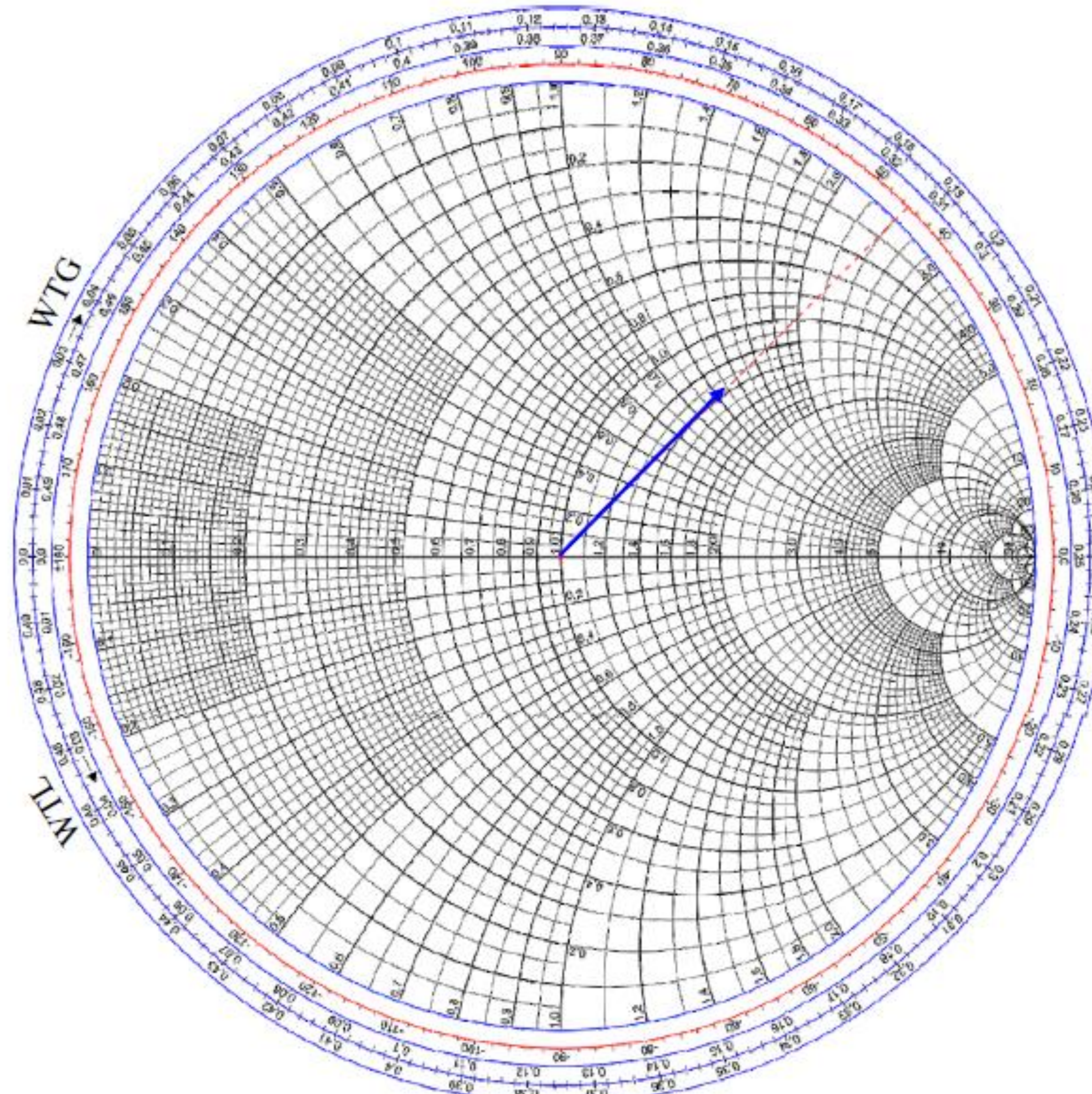
Given:

$$\Gamma_L = 0.5\angle 45°$$

$$Z_o = 50\Omega$$

What is Z_L?

$$Z_L = 50\Omega(1.39 + j1.35)$$
$$= 69.5\Omega + j67.5\Omega$$



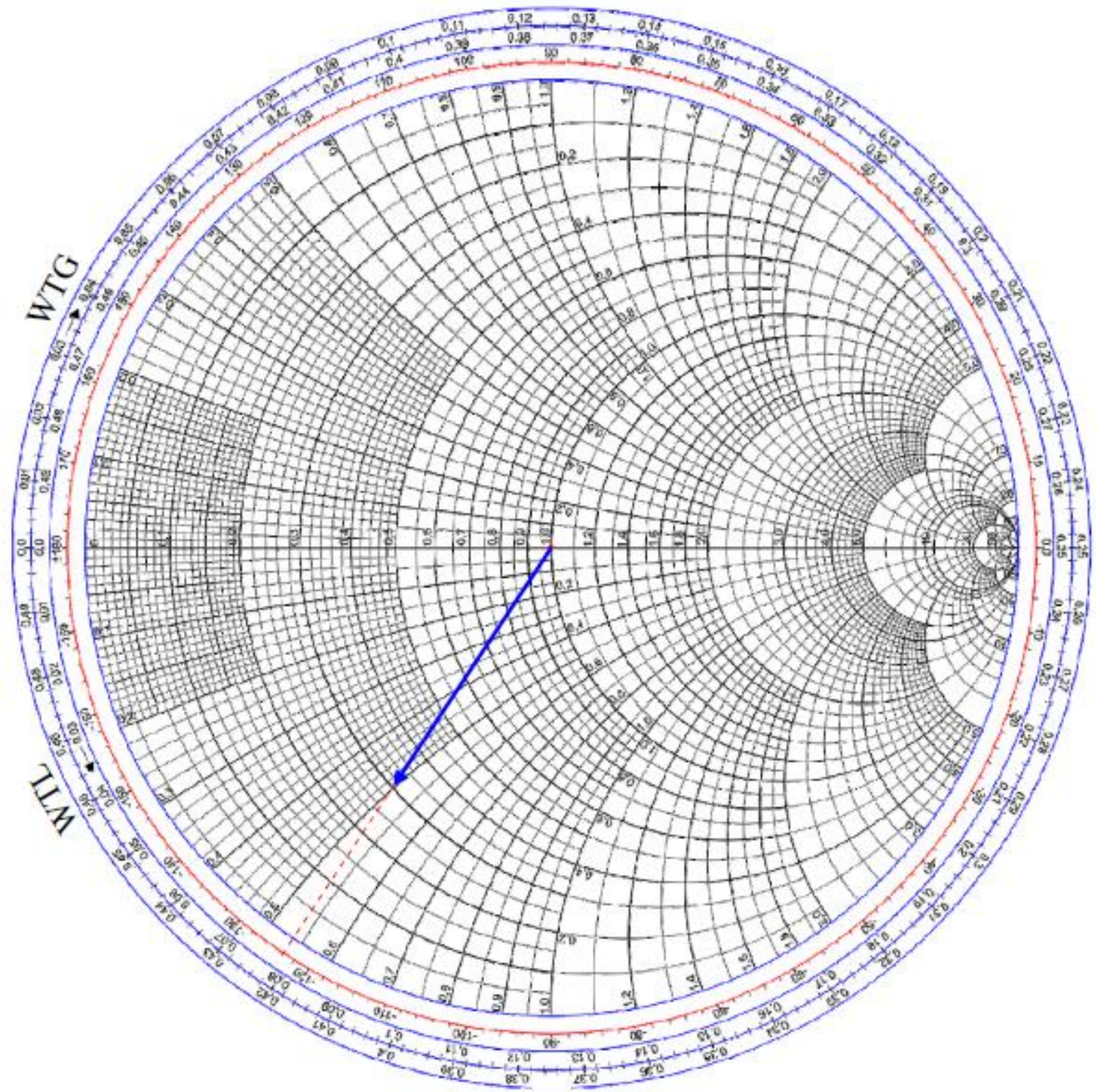
Smith Chart Example 2

Given:

$$Z_L = 15\Omega - j25\Omega$$
$$Z_o = 50\Omega$$

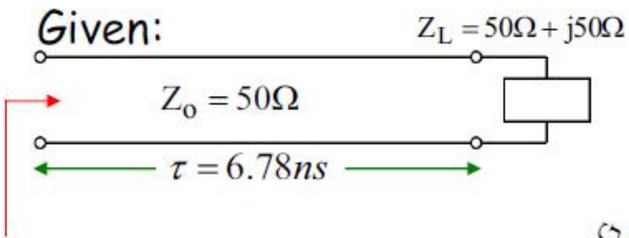
What is Γ_L?

$$z_L = \frac{15\Omega - j25\Omega}{50\Omega}$$
$$= 0.3 - j0.5$$
$$\Gamma_L = 0.6\angle -123°$$



Smith Chart Example 3

Given:



What is Z_{in} at 50 MHz?

$$z_L = \frac{50\Omega + j50\Omega}{50\Omega}$$

$$= 1.0 + j1.0$$

$$\Gamma_L = 0.445\angle 64°$$

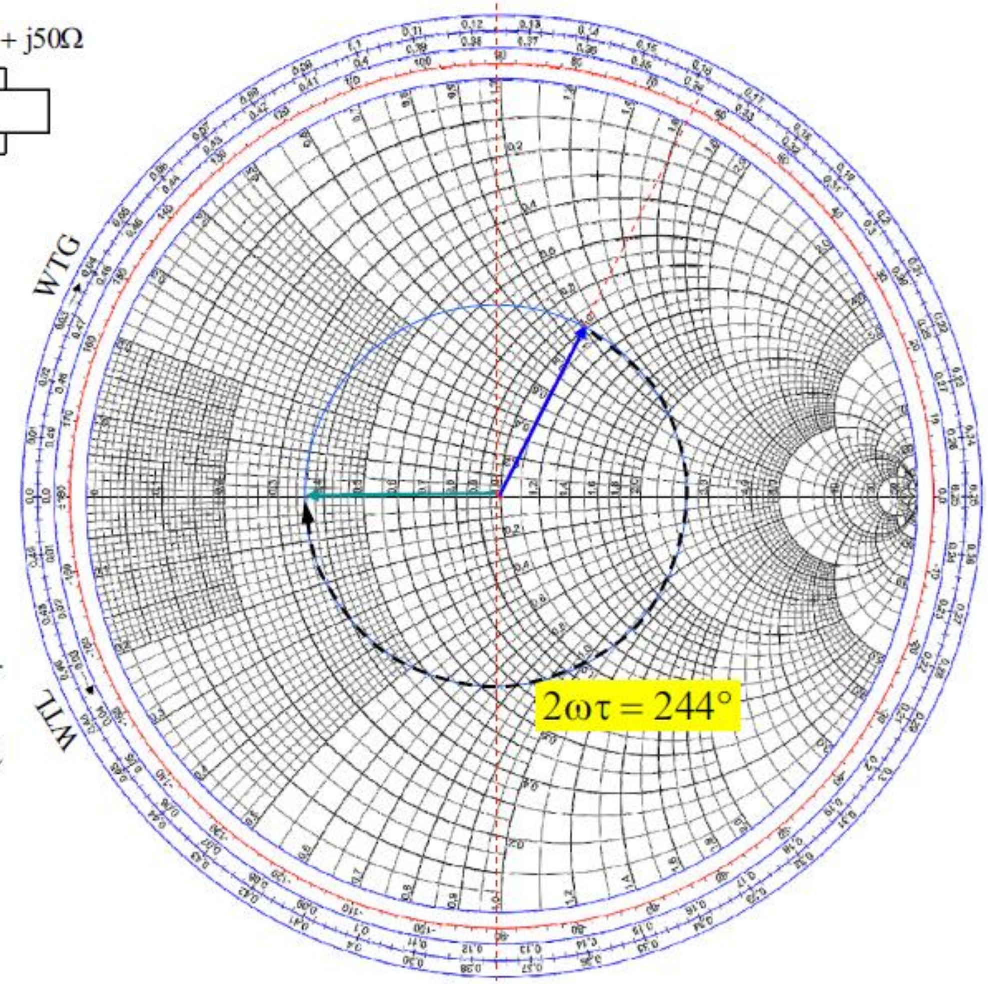
$$\Gamma_{in} = \Gamma_L e^{-j2\beta l} = \Gamma_L e^{-j4\pi l/\lambda} = \Gamma_L e^{-j2\omega\tau}$$

$$l = f\lambda\tau = 50\cdot 10^6 \cdot 6.78\cdot 10^{-9}\lambda = 0.339\lambda$$

$$\vartheta_{in} = 180°$$

$$\Gamma_{in} = 0.445\angle 180°$$

$$Z_{in} = 50\Omega(0.38 + j0.0) = 19\Omega$$



Peryod: $T=1/f=20 \times 10^{-9}$ sec

Hattın uzunluğu: $\tau=6.78 \times 10^{-9}$ sec verildiğinde

$k\lambda$ yı bulmak için k nın belirlenmesi gerekir. $k= \tau/T =0.339$ bulunur.

Kaynaklar

- https://www.ece.ucsb.edu/~long/ece145a/ampdesign.pdf
- Amplifiers, Prof. Tzong-Lin Wu. EMC Laboratory. Department of Electrical Engineering. National Taiwan University

Usage Notes

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Sincerely,

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